Homework 19: Double integrals

This homework is due Wednesday, 10/28 rsp Thursday 10/29.

1 a) (4 points) Find the iterated integral

$$\int_0^1 \int_0^2 6xy/\sqrt{x^2+2y^2}\, dy\, dx\ .$$

b) (4 points) Now compute

$$\int_0^1 \int_0^2 6xy/\sqrt{x^2+2y^2}\, dx\, dy\ .$$

c) (2 points) Wouldn't Fubini assure that a) and b) are the same? What change would be needed in b) to make the results agree?

2 Calculate the double integral

$$\int\int_R \frac{6x}{1+xy}\, dA$$

over the region $R = \{(x,y) \mid 0 \le y \le x, 0 \le x \le 1\}$.

3 a) Evaluate the double integral

$$\int_0^2 \int_y^{2y} 3xy\, dx\, dy\ .$$

b) Evaluate the integral of xy over the same region but as a type I integral (which needs to be split into two integrals):

$$\int_0^2 \int_{\dots}^{\dots} 3xy\ dydx + \int_2^4 \int_{\dots}^{\dots} 3xy\ dydx\ .$$

4 We want to find the volume of the region below the graph of $f(x,y) = 15xy^2$, where R is the region enclosed by the curves $x = 0, x = \sqrt{1-y^2}$. In other words, find $\int\int_R 15xy^2\, dA$.

5 Evaluate the integral:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 4e^{x^4}\, dx\, dy\ .$$

Main definitions

If R is a two dimensional region and $f(x,y)$ a function of two variables, the **double integral** $\int\int_R f(x,y)\, dA$ is defined the limit of the Riemann sum $(1/n^2)\sum_{(i/n,j/n)\in R} f(i/n,j/n)$ for $n \to \infty$. Depending on how we order the sum we write $dA = dxdy$ or $dA = dydx$ and reduce to single variable integrals.

A **type I region** is of the form

$$R = \{(x,y) \mid a \leq x \leq b,\ c(x) \leq y \leq d(x)\ \}\ .$$

An integral over such a region is called a **type I integral**

$$\iint_R f\ dA = \int_a^b \int_{c(x)}^{d(x)} f(x,y)\ dydx\ .$$

A **type II region** is of the form

$$R = \{(x,y) \mid c \leq y \leq d, a(y) \leq x \leq b(y)\ \}\ .$$

An integral over such a region is called a **type II integral**

$$\iint_R f\ dA = \int_c^d \int_{a(y)}^{b(y)} f(x,y)\ dxdy\ .$$

Fubini's theorem allows to switch the order of integration over a rectangle, if the function f is continuous:

$$\int_a^b \int_c^d f(x,y)\ dxdy = \int_c^d \int_a^b f(x,y)\ dydx\ .$$