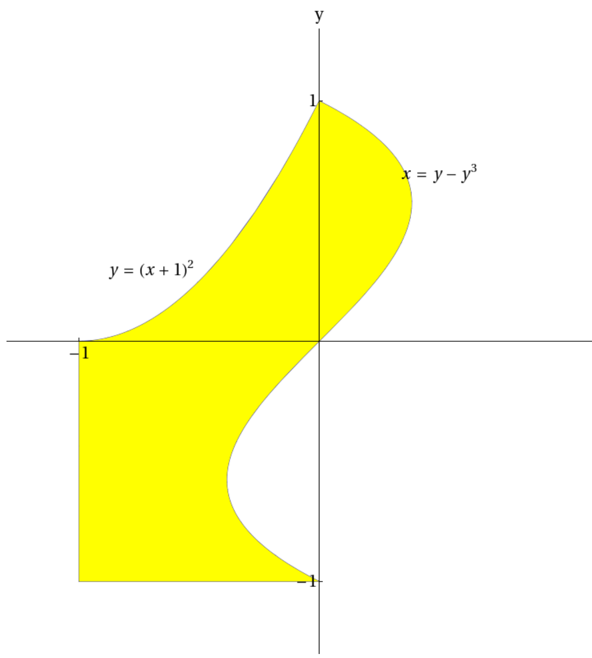
Homework 20: Polar integration

This homework is due Friday, 10/30 rsp Tuesday 11/3.

1 This is a review problem from the last section which does not deal with polar integration yet. Express the region R bound by the four curves $x = -1, y = -1, y = (x+1)^2, x = y - y^3$ as a union of type I or type II regions and evaluate the integral.

$$\iint_R y \, dA \; .$$



2 Evaluate the given integral by changing to polar coordinates:

$$\iint_R 6x \, dA \; ,$$

where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

3 Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

4 Let D be the disk with center the origin and radius a. What is the average distance from points in D to the origin?

5 Evaluate the iterated integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} 9\sqrt{x^2+y^2} \; dy \, dx \; .$$

Main definitions

Polar coordinates $(x, y) = (r\cos(t), r\sin(t))$ allow to describe regions bound by polar curves $(r(\theta), \theta)$.

The **average** of a quantity $f(x, y)$ over a region G is the fraction

$$\frac{\int\int_G f(x,y)\ dA}{\int\int_G 1\ dA}\ .$$

To integrate in polar coordinates, we evaluate the integral

$$\int\int_R f(x,y)\ dxdy = \int\int_R f(r\cos(\theta), r\sin(\theta))r\ drd\theta$$

where R is described in polar coordinates.