

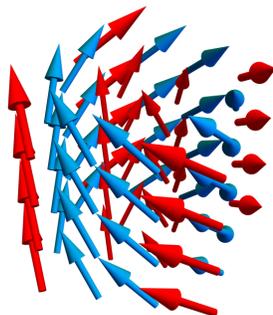
Homework 24: Vector fields

This homework is due Wednesday, 11/11 rsp Thursday 11/12.

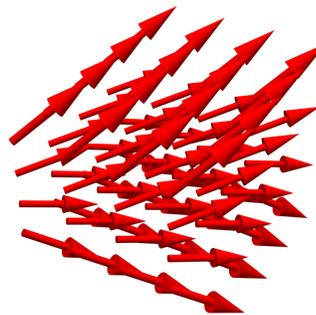
1 Match the vector fields \vec{F} with the plots labeled A-D.

a) $\vec{F}(x, y, z) = \langle 0, y, z \rangle$, b) $\vec{F}(x, y, z) = \langle y, -x, 1 \rangle$, c) $\vec{F}(x, y, z) = \langle 1, 1, z \rangle$

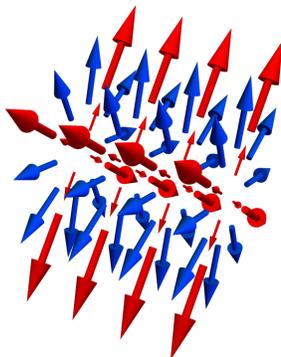
d) $\vec{F}(x, y, z) = \langle x, y, z \rangle$



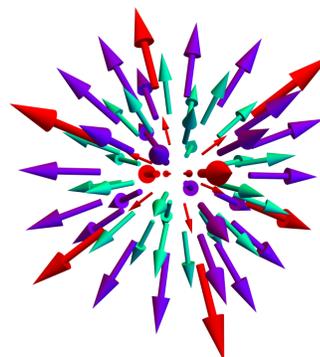
A



B



C



D

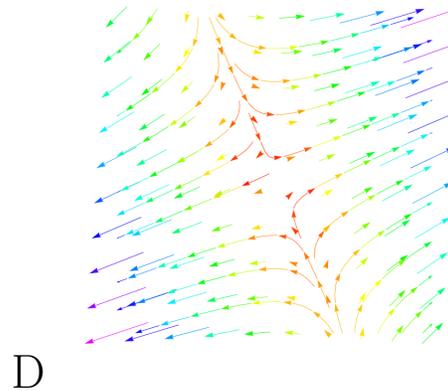
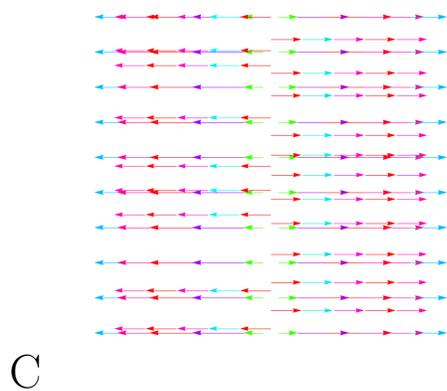
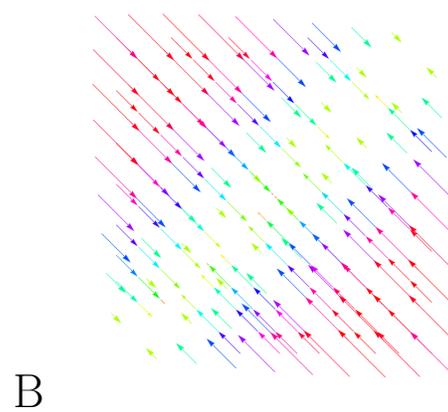
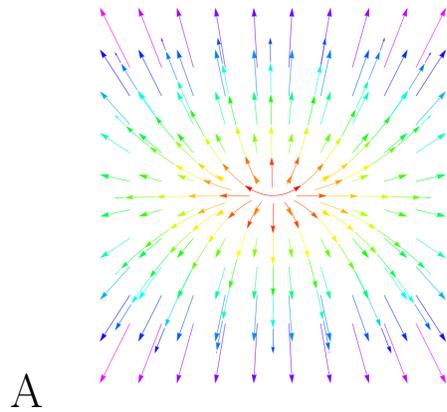
2 a) Compute gradient vector field of f , where $f(x, y) = \log(7x - 5y)$ and \log is the natural log.

b) Given the vector field $\vec{F} = \langle P, Q \rangle = \left\langle \frac{x}{\sqrt{x^2 - 3y^2}} + 3y + 1, 3x - \frac{3y}{\sqrt{x^2 - 3y^2}} \right\rangle$. Check that $Q_x - P_y = 0$ and find a function $f(x, y)$ for which $\nabla f = \vec{F}$.

3 Match the functions f with the plots of their gradient fields labeled $A - D$. Give reasons for your choices.

a) $f(x, y) = \cos(x - y)$, b) $f(x, y) = \sin(\sqrt{1 + x^2})$

c) $f(x, y) = x^2 + 2y^2$, d) $f(x, y) = x(x + y)$



4 a) Sketch the vector field $\vec{F}(x, y) = \langle 2, x \rangle$ and then sketch some flow lines. What shape to these flow lines appear to have?

b) Find the flow line $\vec{r}(t)$ with $\vec{r}(0) = \langle 0, 0 \rangle$.

5 a) Let $\vec{F}(x, y) = \langle x, y \rangle (r^2 - 2r)$ with $r = |\langle x, y \rangle|$. Plot the vector field using Mathematica.

b) Make a stream plot of the field $\vec{F}(x, y) = \langle x^2y, x + y \rangle$ using Mathematica. Example code is below.

```
VectorPlot[{x+y, x^2}, {x, 1, 2}, {y, -1, 1}]
StreamPlot[{x+y, x^2}, {x, 1, 2}, {y, -1, 1}]
VectorPlot3D[{x, y, z}, {x, 1, 2}, {y, 1, 2}, {z, 1, 2}]
```