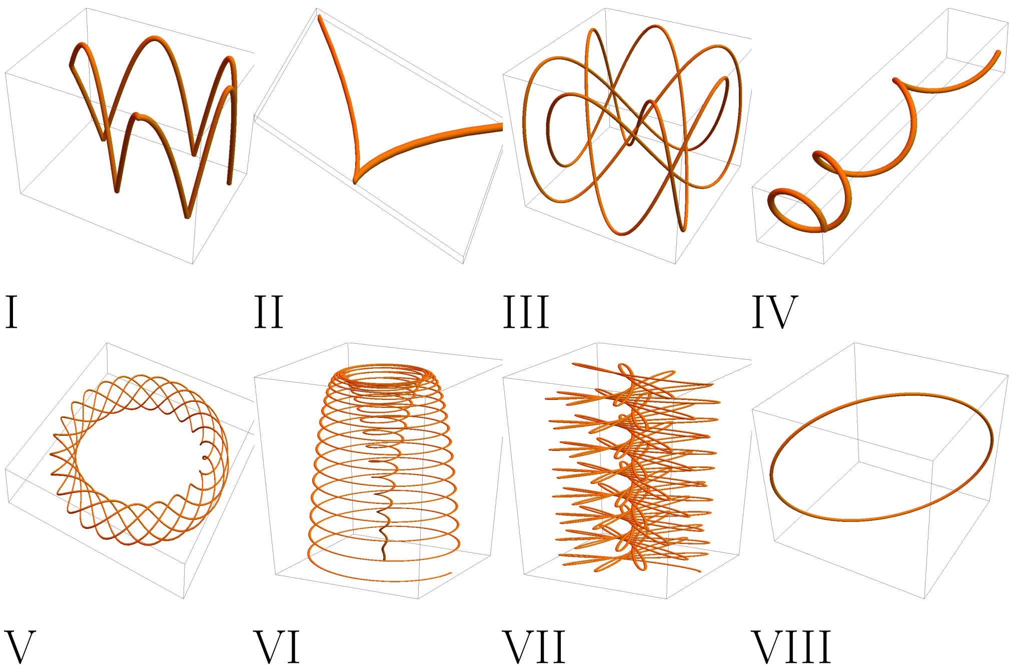
Homework 5: Parametrized curves

This homework is due Monday, 9/21 rsp Tuesday 9/22.

1 Match the curves:



$\vec{r}(t) =$	I-VIII
$\langle t\cos(8t), t\sin(8t), t(8\pi - t)\rangle$	
$\langle (6+\cos(24t))\cos(5t), (6+\cos(24t])\sin(5t), \sin(24t)\rangle$	
$\langle \cos(t40)\cos(3t), \cos(t40)\sin(4t), t\rangle$	
$\langle \cos(t), t^2, \sin(t)\rangle$	
$\langle t^3, t^2, 0\rangle$	
$\langle \cos(3t), \sin(4t), \cos(7t)\rangle$	
$\langle \cos(t), \cos(t), \sin(t)\rangle$	
$\langle \vert\cos(t)+\sin(t)\vert, \vert\sin(t)\vert, \vert\cos(5t)\vert\rangle$	

2 Parametrize the intersection of the elliptic paraboloid $z = 2x^2 + y^2$ with the quintic cylinder $y = 1 + x^5$.

3 a) Two particles travel along the space curves $\vec{r}_1(t) = \langle t, t^2, t^3\rangle$ and $\vec{r}_2(t) = \langle 1+2t, 1+6t, 1+14t\rangle$. Do the particles collide? Do the particle paths intersect?

b) If $\vec{r}(t) = \langle \cos(t), 2\sin(t), 4t \rangle$, find $\vec{r}\,''(0)$ and $\vec{r}\,'(0)$. Then compute $|\vec{r}\,'(0) \times \vec{r}\,''(0)|/|\vec{r}\,'(0)|^3$. We will later call this the curvature.

4 Find the point of intersection of the two tangent lines to the curve $\vec{r}(t) = \langle \sin(\pi t), 2\sin(\pi t), \cos(\pi t) \rangle$ at the points where $t = 0$ and $t = 0.5$.

5 A particle moving along a curve $\vec{r}(t)$ has the property that $\vec{r}\,''(t) = \langle 1, 0, \sin(2t) \rangle$. We know $\vec{r}(0) = \langle 1, 1, 2 \rangle$ and $\vec{r}\,'(0) = \langle 1, 0, 0 \rangle$. What is $\vec{r}(\pi)$?

Main definitions

A **parametrization** of a planar curve is a map $\vec{r}(t) = \langle x(t), y(t) \rangle$ from a **parameter interval** $R = [a, b]$ to the plane. The functions $x(t), y(t)$ are called **coordinate functions**. The image of the parametrization is called a **parametrized curve** in the plane. The parametrization of a space curve is $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$. The **image** of r is a **parametrized curve** in space.

If $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a curve, then $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle = \langle \dot{x}, \dot{y}, \dot{z} \rangle$ is called the **velocity** at time t. Its length $|\vec{r}\,'(t)|$ is called **speed** and $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$ is called **unit tangent vector** or direction of motion. The vector $\vec{r}\,''(t)$ is called the **acceleration**. When knowing the acceleration and $\vec{r}\,'(0)$ and $\vec{r}(0)$ we can get back position $\vec{r}(t)$ by integration. Similarly, if we know $\vec{r}\,''(t)$ at all times and $\vec{r}(0)$ and $\vec{r}\,'(0)$, we can compute $\vec{r}(t)$ by integration.