

Homework 7: Other coordinates

This homework is due Friday, 9/25 resp Tuesday 9/29. (Tue/Thu sections: Do it early!)

- 1 Change the following points to cylindrical coordinates: a) $(2\sqrt{3}, 2, 17)$
 b) $(4, -3, 6)$
 Change to spherical coordinates: c) $(0, \sqrt{3}, 1)$ d) $(-1, 1, \sqrt{6})$

- 2 a) Identify the surface $r^2 = (z - 5)^2$.
 b) Identify the surface given in spherical coordinates as $\rho \sin(\phi) = 2$.

- 3 a) Identify the surface $2r^2 - z^2 = 1$.
 b) Identify the surface $\cos^2(\phi) = \sin^2(\phi)$.

- 4 a) Identify the surface whose equation is given in cylindrical coordinates as

$$\cos(\theta) + \sin(\theta) = 1/r .$$

- b) Identify the surface whose equation is given in spherical coordinates as

$$\rho^2(\sin^2(\phi) \sin^2(\theta) + \cos^2(\phi)) = 16 .$$

- 5 a) Use the Mathematica command "RevolutionPlot3D" to plot the surface which is given by

$$z = (\sin(5\theta) + \cos(5\theta))e^{-r^2} .$$

- (See example Mathematica command below). b) Use the Mathematica command "SphericalPlot3D" to plot a bumpy sphere

$$\rho(\phi, \theta) = (3 + \cos(\theta) + \sin(11\theta) \sin(13\phi)) .$$

with $0 \leq \phi \leq \pi$ and $0 \leq \theta < 2\pi$.

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RevolutionPlot3D[Cos[r t], {r, 0, Pi}, {t, 0, 2 Pi}]
SphericalPlot3D[s+ t, {s, 0, Pi}, {t, 0, 2 Pi}]
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Main definitions

A point (x, y) in the plane has the **polar coordinates** $r = \sqrt{x^2 + y^2} \geq 0, \theta = \text{arctg}(y/x)$. We have the relation $(x, y) = (r \cos(\theta), r \sin(\theta))$. Chose the arctan values in $(-\pi/2, \pi/2]$ and add π if $x < 0$ or $x = 0, y < 0$. A point (x, y, z) in space has the **cylindrical coordinates** (r, θ, z) , where (r, θ) are the polar coordinates of (x, y) .

A curve given in polar coordinates as $r(\theta) = f(\theta)$ is called a **polar curve**. It can in Cartesian coordinates be described as $\vec{r}(t) = \langle f(t) \cos(t), f(t) \sin(t) \rangle$.

Spherical coordinates use the distance ρ to the origin as well as two angles θ and ϕ . The first angle θ is the polar angle in polar coordinates of the xy coordinates and ϕ is the angle between the vector \vec{OP} and the z -axis. The relation is $(x, y, z) = (\rho \cos(\theta) \sin(\phi), \rho \sin(\theta) \sin(\phi), \rho \cos(\phi))$. We have

$$\begin{aligned}x &= \rho \cos(\theta) \sin(\phi), \\y &= \rho \sin(\theta) \sin(\phi), \\z &= \rho \cos(\phi)\end{aligned}$$