Name:

MWF 9 Koji Shimizu
MWF 10 Can Kozcaz
MWF 10 Yifei Zhao
MWF 11 Oliver Knill
MWF 11 Bena Tshishiku
MWF 12 Jun-Hou Fung
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TTH 10 Jameel Al-Aidroos
TTH 10 Ziliang Che
TTH 10 George Melvin
TTH 11:30 Jake Marcinek
TTH 11:30 George Melvin

- Start by printing your name in the above box and please **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, slide rules, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

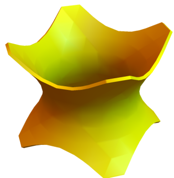
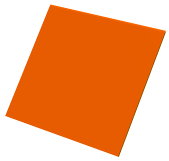
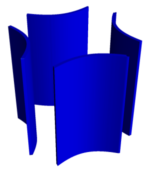
Problem 1) (20 points) No justifications are needed.

1)	T	F	If $\|\vec{r}\,'(t)\| = 1$ for all t, then the curvature satisfies $\kappa(t) = \|\vec{r}\,''(t)\|$.
2)	T	F	Let $\vec{r}(t)$ be a parametric curve. Suppose that at the point $\vec{r}(t)$ the unit tangent vector is $\langle 0,1,0 \rangle$ and the binormal vector is $\langle 0,0,1 \rangle$. Then the unit normal vector at the point is $\langle 1,0,0 \rangle$.
3)	T	F	If $\|\vec{v}+\vec{w}\|^2 = 2\vec{v}\cdot\vec{w}$, then $\vec{v} = 2\vec{w}$.
4)	T	F	The distance between two points P and Q is smaller than or equal to $\|\vec{OP}\| + \|\vec{OQ}\|$, where $O = (0,0,0)$ is the origin.
5)	T	F	The line $\vec{r}(t) = \langle 5+2t, 2+t, 3+t \rangle$ is located on the plane $x-y-z=0$.
6)	T	F	The arc length of a circle with constant curvature 20 is $\pi/10$.
7)	T	F	The surface $x^2 - y^2 + 4y = z^2 + 2z$ is an elliptic paraboloid.
8)	T	F	For any two vectors, $\|\vec{v}\cdot\vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$.
9)	T	F	The acceleration of $\vec{r}(t) = \langle t,t,t \rangle$ is $\langle 0,0,0 \rangle$ everywhere.
10)	T	F	If $\vec{v} = \vec{PQ} = \langle 2,1,1 \rangle$ then $\|\vec{v}\|$ is larger than the distance between P and Q.
11)	T	F	If $\vec{v}\cdot\vec{w} > 0$, then the angle between $\vec{v}$ and $\vec{w}$ is larger than $90°$.
12)	T	F	If $\langle 5,6,4 \rangle \times \vec{x} = \vec{x}$ then $\vec{x} = \langle 0,0,0 \rangle$.
13)	T	F	The curvature of $\vec{r}(t) = \langle t-1, 1-t, t \rangle$ is 0 at the point $t=1$.
14)	T	F	The line $\vec{r}(t) = t\langle 7,7,1 \rangle$ hits the plane $-x-y=7z$ in a right angle.
15)	T	F	The surface given in spherical coordinates as $\sin(\phi) = 1/\rho$ is a cylinder.
16)	T	F	Given three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, then $\|(\vec{u}\cdot\vec{v})(\vec{w}\cdot\vec{u})\| \leq \|\vec{u}\|^2\|\vec{v}\|\|\vec{w}\|$.
17)	T	F	The surface given in spherical coordinates as $\rho^2 = 1$ is a sphere.
18)	T	F	The arc length of the curve $\langle 5\sin(t), 1, 5\cos(t) \rangle$ from $t=0$ to $t=2\pi$ is equal to 10π.
19)	T	F	The surface parametrized by $\vec{r}(u,v) = \langle u^5 - v^5, u^5 + v^5, u^5 \rangle$ is a plane.
20)	T	F	If the non-zero cross product of a vector $\vec{v}$ with a vector $\vec{w}$ is parallel to $\vec{v}$, then the dot product between $\vec{v}$ and $\vec{w}$ is zero.

	Total

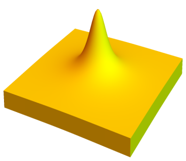
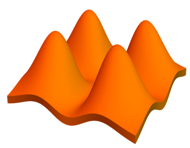
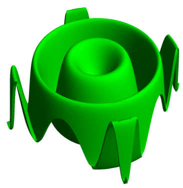
Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the contour surfaces $g(x, y, z) = 1$. Enter O, if there is no match.

I  II  III 

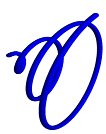
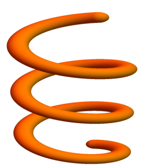
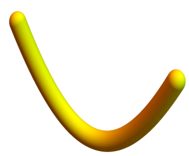
Function $g(x, y, z) = 1$	Enter O,I,II or III
$x^2 + y^2 - z^2$	
x^2y^2	
$x - y$	
$x^2 + z^2$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.

I  II  III 

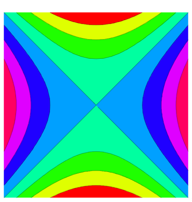
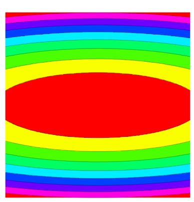
Function $f(x, y) =$	Enter O,I,II or III
$1/(1 + x^2 + y^2)$	
$x^2y^2e^{-x^2-y^2}$	
$\sin(x^2 + y^2)$	
$\cos(y)$	

c) (2 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.

I  II  III 

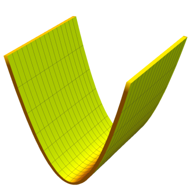
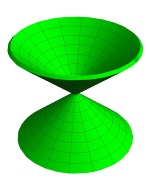
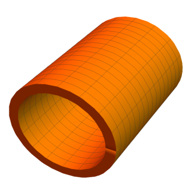
Parametrization $\vec{r}(t) =$	Enter O, I,II or III
$\langle t, t\sin(3t), t\cos(3t)\rangle$	
$\langle \cos(t), \sin(t), \cos(t)\rangle$	
$\langle \sin(t), \cos(t), t\rangle$	
$\langle t, t, t^2\rangle$	

d) (2 points) Match functions g with contour plots in the xy-plane. Enter O, if there is no match.

I  II  III 

Equation	Enter O, I,II or III
$\sin(x) - y$	
$x^2 - y^2$	
$10x^2 + y^2$	
$x^2 + 10y^2$	

e) (2 points) Match the quadrics. Enter O if no match.

I  II  III 

Quadric	Enter O,I,II or III
$x^2 = z$	
$x^2 + y^2 = 1$	
$xy = 1$	
$y = 1$	
$x^2 + y^2 = z^2$	

Problem 3) (10 points)	(Only answers are needed)

a) (4 points) Mark what applies for any two vectors $\vec{v}$ and $\vec{w}$ in space.

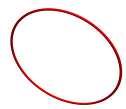
Object	always 0	can be $\neq 0$	always $\vec{0} = \langle 0,0,0 \rangle$	can be nonzero vector
$(\vec{v} \times \vec{w}) \times \vec{v}$				
$(\vec{v} \times \vec{w}) \cdot \vec{v}$				
$(\text{proj}_{\vec{v}}\vec{w}) \times \vec{w}$				
$(\text{proj}_{\vec{w}}\vec{v}) \cdot \vec{w}$				

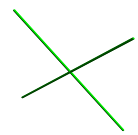
b) (3 points) **Conic sections** (parabolas, ellipses or hyperbolas) can be seen as intersections of a two dimensional cone

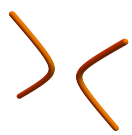
$$x^2 + y^2 = z^2$$

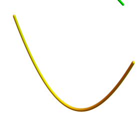
with a 2D plane. Identify the quadrics in the following three cases:

Intersect with plane	Enter A-D
$z = 1$	
$z = \sqrt{2}x$	
$x = y + 1$	

A 

B 

C 

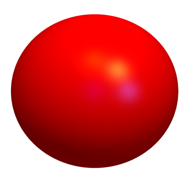
D 

c) (3 points) Three dimensional cone is given by the equation

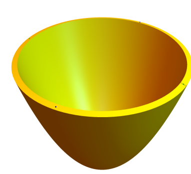
$$x^2 + y^2 + z^2 = w^2$$

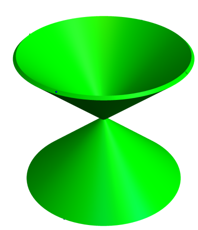
in four dimensional space. If we intersect it with a three dimensional space, we get quadrics. We want you to identify a few quadrics. In the pictures the quadrics might be turned or scaled. You get a point for every right answer, meaning that you can miss one and still have full credit.

Intersect with the 3D plane	Enter A-D
$w = x + y$	
$w = 2z + 1$	
$y = z$	
$w = 1$	

A 

B 

C 

D 

Problem 4) (10 points)

We are going into the **furniture design business**. Our first task is to construct a chair. Find the distance between the lines spanned by the parallel arm rests AB and CD, where

$$A = (1, 0, 2), B = (4, 1, 2)$$

and

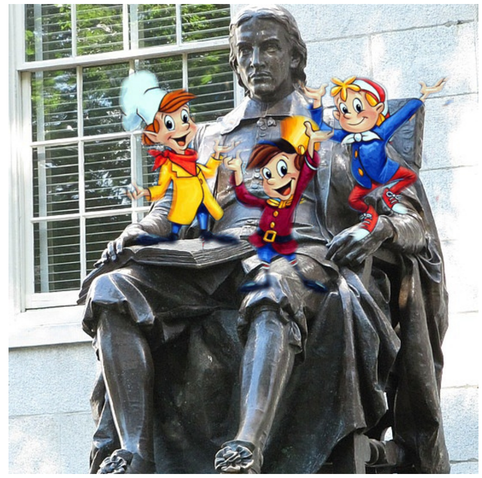
$$C = (3, 3, 3), D = (6, 4, 3) \ .$$

Problem 5) (10 points)

a) (6 points) The first derivative of parametrized curve is called **"velocity"**. You have also learned the terms **"acceleration"** and maybe **"jerk"** for the second and third derivative. Less well known are **"snap"**, **"crackle"**, **"pop"** for the fourth, fifth and sixth derivatives. Since we could not yet find the seventh derivative named, let's call it the **"Harvard"**. Compute the **"Harvard"** of the curve

$$\vec{r}(t) = \langle \cos(2t) + t, \sin(2t), t^2 \rangle$$

at time $t = 0$.



b) (4 points) The parametrization $\vec{v}(t) = \vec{r}\,'(t)$ defines a new curve. It is located on a surface. Which of the following surface is it?

	Check one
cylinder	A
cone	B
plane	C
ellipsoid	D
paraboloid	E

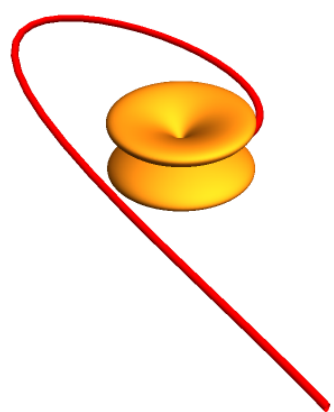
Problem 6) (10 points)

A kid plays with a **Yo-Yo**. It is accelerated periodically with $\vec{r}\,''(t) = \langle \sin(t), 0, \cos(t) - 10 \rangle$. Find the position of the Yo-Yo at time $t = 2\pi$ if the initial position is

$$\vec{r}(0) = \langle 5, 5, 0 \rangle$$

and the initial velocity is

$$\vec{r}\,'(0) = \langle 1, 1, 1 \rangle \ .$$

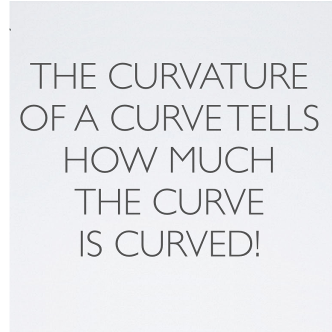


Problem 7) (10 points)

a) (5 points) Compute the arc length of the curve

$$\vec{r}(t) = \langle t, t^2, 2t^3/3 \rangle$$

if $0 \leq t \leq 4$.



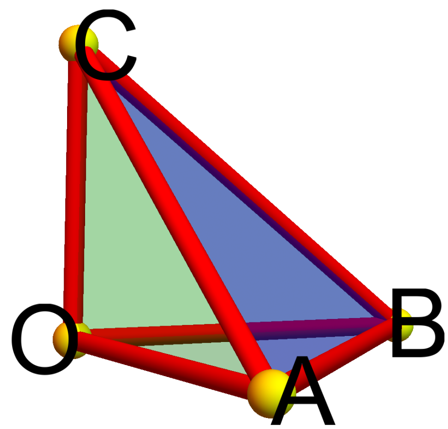
b) (5 points) What is the curvature

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

of this curve at $t = 0$?

Problem 8) (10 points)

The triangle ABC is obtained by slicing a corner ABC off from a cube. One obtains a so called **trirectangular tetrahedron**.



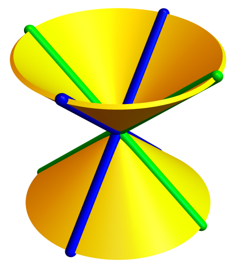
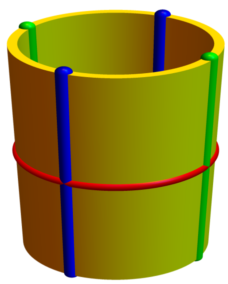
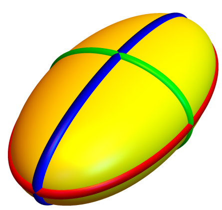
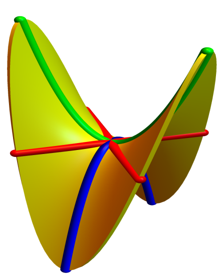
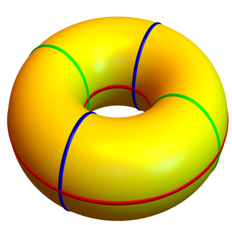
a) (5 points) Find the square of the area of the triangle ABC, where $A = (3, 0, 0), B = (0, 6, 0), C = (0, 0, 8)$.

b) (5 points) Compute also the sum of the squares of the areas of the triangles OAB, OBC and OCA, where $O = (0, 0, 0)$ is the origin. The sum should get the same value you got in a).

P.S. the same computation can be repeated for arbitrary points $A = (a, 0, 0), B = (0, b, 0), C = (0, 0, c)$. It proves a not so well known theorem telling that the sum of the squares of the side wall areas is the square of the face area. It is a 3 dimensional version of Pythagoras and also goes by the name **de Gua theorem** or **Faulhaber theorem**.

Problem 9) (10 points)

The 3D printing venture "Math-Candy" (math-candy.com) asks you to do some product development. In each of the 5 following parametrizations, two entries are still missing, each entry being worth one candy (1 point).

a)		The surface $\boxed{x^2+y^2=z^2}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots\ldots\ldots , \ldots\ldots\ldots\ldots\ldots , z\rangle$
b)		The surface $\boxed{x^2+y^2=1}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots\ldots , \ldots\ldots\ldots\ldots , z\rangle$
c)		The surface $\boxed{2(x-1)^2+(y-5)^2+4z^2=1}$ is parametrized by $\vec{r}(\theta,\phi)=\langle \ldots\ldots\ldots\ldots , \ldots\ldots\ldots\ldots , \cos(\phi)/2\rangle$
d)		The surface $\boxed{x^2-y^2=z}$ is parametrized by $\vec{r}(x,y)=\langle \ldots\ldots\ldots , \ldots\ldots\ldots , x^2-y^2\rangle$
e)		The surface $\boxed{(\sqrt{x^2+y^2}-2)^2+z^2=1}$ is parametrized by $\vec{r}(\theta,\phi)=\langle (2+\cos(\phi))\cos(\theta), \ldots\ldots\ldots\ldots , \ldots\ldots\ldots \rangle$