

Name:

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TTH 11:30 Jake Marcinek
TTH 11:30 George Melvin

- Start by printing your name in the above box and please **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, slide rules, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes to complete your work.

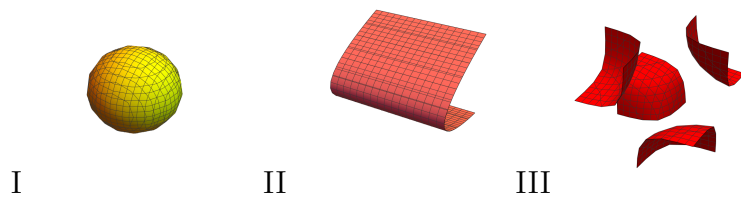
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

- 1) ☐ T ☐ F The vector $\langle 2, 3, 6 \rangle$ has a length which is an integer.
- 2) ☐ T ☐ F The surface $x^2 + y^2 + z^2 - 2x = 3$ is a sphere.
- 3) ☐ T ☐ F If $\vec{v} \cdot \vec{w}$ is negative, then the angle between $\vec{v}$ and $\vec{w}$ is acute (= smaller than $\pi/2$).
- 4) ☐ T ☐ F The level curves $f(x, y) = 1$ and $f(x, y) = 0$ do not intersect for $f(x, y) = (xy + \cos(x))^6$.
- 5) ☐ T ☐ F For any nonzero $\vec{a}$, the equation $\vec{a} \times \vec{x} = \vec{b}$ always has a solution $\vec{x}$.
- 6) ☐ T ☐ F For two non-parallel $\vec{a}, \vec{b}$, the equation $(\langle x, y, z \rangle \times \vec{a}) \cdot \vec{b} = 1$ defines a plane.
- 7) ☐ T ☐ F The curvature of $\vec{r}(t) = \langle t^3, 1 - t^3, t^3 \rangle$ is 0 if $t = 1$.
- 8) ☐ T ☐ F If $\vec{N}$ is the normal vector and $\vec{T}$ the unit tangent vector to a curve $\vec{r}(t)$ then the vector projection of $\vec{N}(t)$ onto $\vec{T}(t)$ is zero.
- 9) ☐ T ☐ F There exist non-parallel vectors $\vec{v}, \vec{w}$ such that $\vec{v} \cdot (\vec{v} \times \vec{w}) = 0$.
- 10) ☐ T ☐ F The point given in spherical coordinates as $\rho = 3, \phi = \pi/2, \theta = \pi$ is on the x -axes.
- 11) ☐ T ☐ F The parametrized curve $\vec{r}(t) = \langle 5 \cos(3t), 3 \sin(3t), 0 \rangle$ is an ellipse.
- 12) ☐ T ☐ F If the vector projection of $\vec{v}$ onto $\vec{w}$ is $\vec{w}$ then $\vec{v} = \vec{w}$.
- 13) ☐ T ☐ F Given three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, then $|(\vec{u} \times \vec{v}) \times \vec{w}| \leq |\vec{u}||\vec{v}||\vec{w}|$.
- 14) ☐ T ☐ F The surface $y^2 + z = x^2$ is a hyperbolic paraboloid.
- 15) ☐ T ☐ F The curvature of a curve $\vec{r}(t)$ at time $t = 0$ is the same as the curvature of $\vec{r}(\sin(t))$ at time $t = 0$.
- 16) ☐ T ☐ F The arc length of the curve $\langle \sin(t), 0, \cos(t) \rangle$ from $t = 0$ to $t = 2\pi$ is equal to 2π .
- 17) ☐ T ☐ F The curve $\vec{r}(t) = \langle \cos(t), \sin(t), \cos(t) + \sin(t) \rangle$ is on the intersection of $x^2 + y^2 = 1$ and $x + y - z = 0$.
- 18) ☐ T ☐ F Using $\vec{i} = \langle 1, 0, 0 \rangle, \vec{j} = \langle 0, 1, 0 \rangle$, the identity $(\vec{i} \times \vec{j}) \times \vec{j} = \vec{i} \times (\vec{j} \times \vec{j})$ holds.
- 19) ☐ T ☐ F Using the same notation, the identity $(\vec{i} \cdot \vec{j})\vec{j} = \vec{i}(\vec{j} \cdot \vec{j})$ holds.
- 20) ☐ T ☐ F $\langle \cos t, \sin t, t \rangle, 0 \leq t \leq 2$ and $\langle \cos(t^3), \sin(t^3), t^3 \rangle, 0 \leq t \leq 2$ have the same arc length.

Total

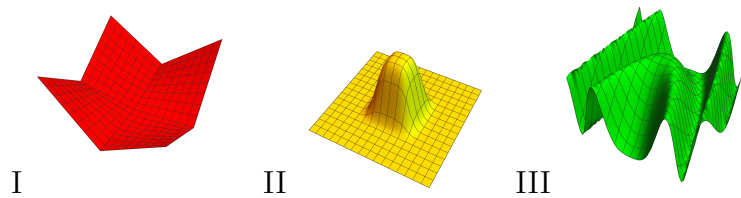
Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the contours $g(x, y, z) = 1$. Enter O, if there is no match.



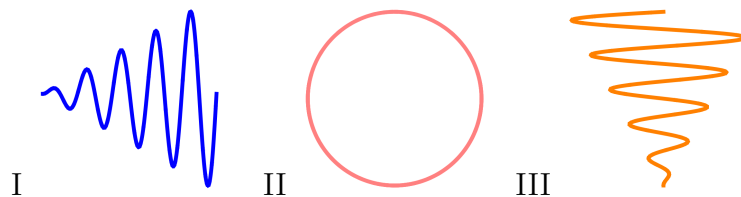
Function $g(x, y, z) = 1$	Enter O,I,II or III
xyz	
$x^2 + y^2 + z^2$	
$z^2 - y$	
$x^2 + z^2$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.



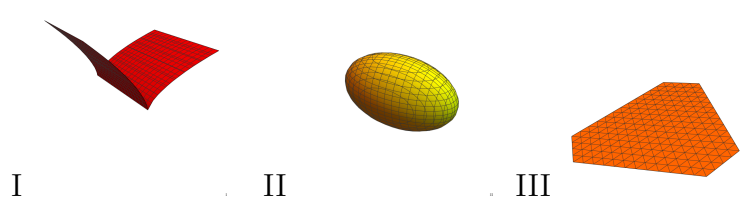
Function $f(x, y) =$	Enter O,I,II or III
$\cos(x^2 + y)$	
$ x + y + xy $	
$\exp(-x^4 - y^4)$	
x^3	

c) (2 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.



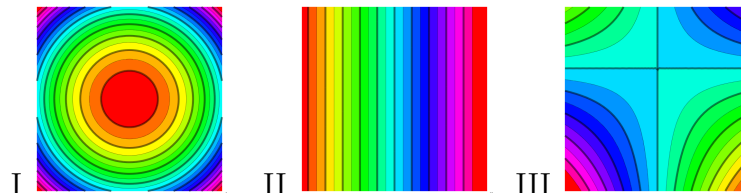
Parametrization $\vec{r}(t) =$	Enter O, I,II or III
$\vec{r}(t) = \langle t, t \sin(5t) \rangle$	
$\vec{r}(t) = \langle t \sin(5t), t \rangle$	
$\vec{r}(t) = \langle \sin(5t), \cos(5t) \rangle$	
$\vec{r}(t) = \langle \cos(5t), \cos(5t) \rangle$	

d) (2 points) Match functions g with level surface $g(x, y, z) = 1$. Enter O, if there is no match.



Function $g(x, y, z) = 1$	Enter O, I,II or III
$x^2 - y^2 + z^2 = 1$	
$x - y - z = 1$	
$y^3 = z^2$	
$x^2/4 + y^2 + z^2/2 = 1$	

e) (2 points) Match the contour maps to a function $f(x, y)$. Enter O if no match.

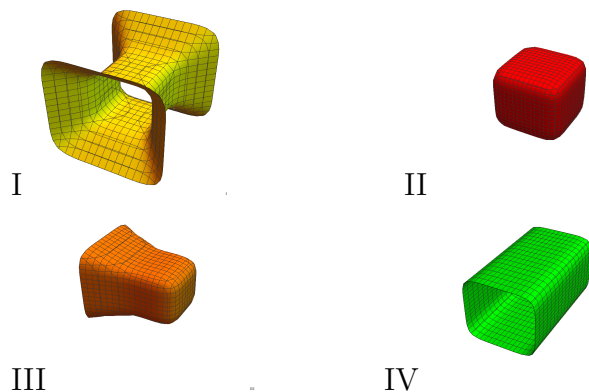


$f(x, y) =$	Enter O,I,II or III
$x^2 - y^4$	
$xy - x$	
x	
y	
$x^2 + y^2$	

Problem 3) (10 points)

(Only answers are needed)

a) (4 points) The following contour surfaces were deformed by setting $X = x^3, Y = y^3, Z = z^3$. Can you label the original quadrics from which it was deformed?



Surface	I-IV	name A-D
$X^2 + Y^2 + Z^2 = 1$		
$X + Y^2 + Z^2 = 1$		
$X^2 - Y^2 + Z^2 = 1$		
$X^2 + Z^2 = 1$		

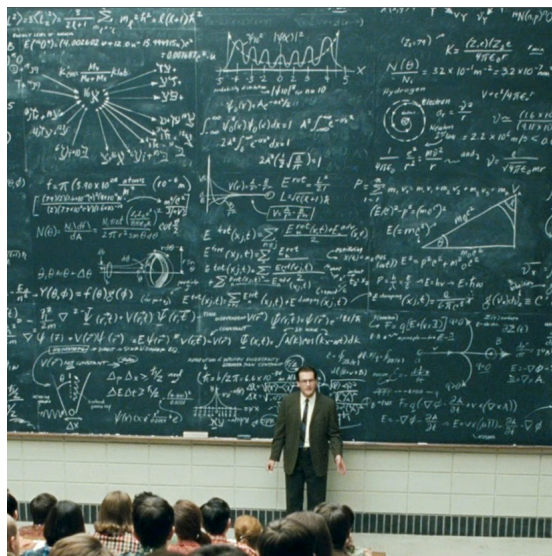
Fill in:

- A) Ellipsoid'esque
- B) Paraboloid'esque
- C) Hyperboloid'esque
- D) Cylinder'esque

b) Help Larry the physicist in the movie "A serious man", to compute some quantities:

$\vec{v} = \langle 1, 2, 3 \rangle$ represent the velocity
 $\vec{\omega} = \langle 0, 1, 1 \rangle$ represent angular velocity
 $\vec{B} = \langle 1, 0, 1 \rangle$ represent magnetic field
 $\vec{r} = \langle 0, 0, 1 \rangle$ represent position. Compute:

- (i) (2 points) Coriolis force $\vec{v} \times \vec{\omega}$.
- (ii) (2 points) Lorentz force $\vec{v} \times \vec{B}$.
- (iii) (1 point) Kinetic energy $(\vec{v} \cdot \vec{v})/2$.
- (iv) (1 point) Magnetic energy $\vec{B} \cdot \vec{B}/2$.



Larry: "I mean - even I don't understand the dead cat. The math is how it really works."

(i)

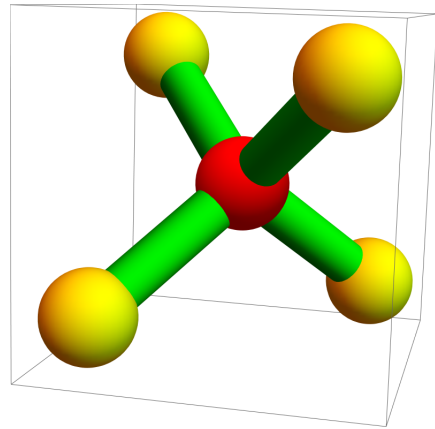
(iii)

ii)

iv)

Problem 4) (10 points)

Methane CH_4 is the number two greenhouse gas emitted by human activity in the US. The four hydrogen atoms of **methane** are located at the vertices $P = (2, 2, 2), Q = (2, 0, 0), R = (0, 2, 0), S = (0, 0, 2)$ and form a regular tetrahedron, while C is the central carbon atom located at $(1, 1, 1)$.



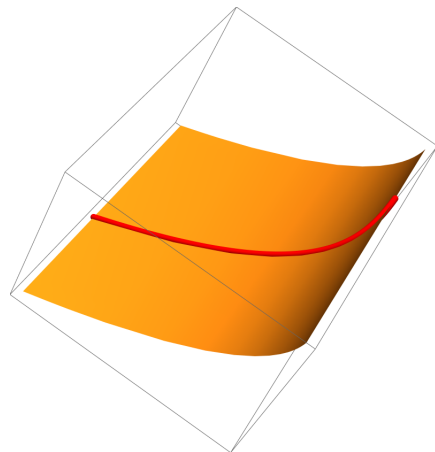
- (2 points) Find one bond distance $|CP|$ and the distance $|PQ|$
- (4 points) Find the cosine of the bond angle between $\overrightarrow{PC}$ and $\overrightarrow{PQ}$.
- (4 points) What is volume of the parallelepiped spanned by $\overrightarrow{PC}, \overrightarrow{QC}, \overrightarrow{RC}$?

Problem 5) (10 points)

Consider the curve

$$\vec{r}(t) = \langle 2e^t, t, e^{2t} \rangle.$$

- (3 points) Compute the speed $|\vec{r}'(0)|$.
- (5 points) Find the arc length from $t = -2$ to $t = 1$.
- (2 points) There exists a constant a such that the curve lies on the cylindrical paraboloid $x^2 = az$. Which a does apply?



Problem 6) (10 points)

The highest **bungee jump** ever recorded was done from the 233 meter high Macau Tower. Assume the rope pulls back with a force $2t$ so that the acceleration is

$$\vec{r}''(t) = \langle 0, 0, 2t - 10 \rangle .$$

Assume the initial velocity is $\langle 1, 0, 0 \rangle$ and that the daredevil jumps from $\vec{r}(0) = \langle 0, 0, 233 \rangle$:

a) (5 points) Find $\vec{r}'(t)$ and determine t_0 for which the third component $z'(t_0) = 0$. This is the time of the lowest point.

b) (5 points) Find $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ and $\vec{r}(t_0)$. Did the jumper hit the ground $z = 0$?



Problem 7) (10 points)

The **logarithmic spiral** is parametrized by $\vec{r}(t) = \langle e^t \cos(t), e^t \sin(t), 0 \rangle$.

a) (5 points) Find the angle between $\vec{r}'(t)$ and acceleration $\vec{r}''(t)$ at time $t = 0$.

b) (5 points) Compute the curvature at $t = 0$.

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} .$$



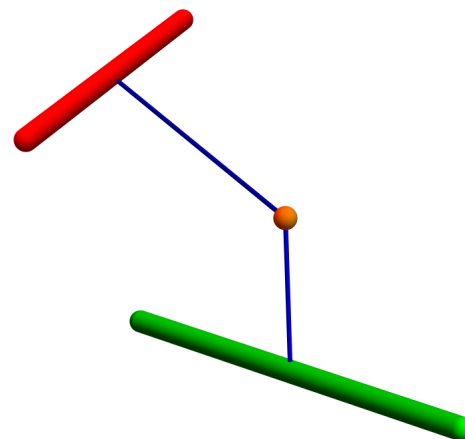
One miracle about the spiral is that arc length from 0 to t multiplied with curvature at t is constant. Jacob Bernoulli called it the curve the “Spira mirabilis” which means “miraculous spiral”.

Problem 8) (10 points)

Given a line $\vec{r}(t) = \langle 1 + t, t, t \rangle$ and a line $\vec{s}(t) = \langle 1 - t, 1 + t, 1 - t \rangle$.

a) (6 points) Find the sum of the distances of the point $(0, 0, 0)$ to the two lines.

b) (4 points) Find the distance between the two lines.



Problem 9) (10 points)

Parametrize the following surfaces in space. As usual r, θ, z are cylindrical and ρ, θ, ϕ are spherical coordinate variables. You do not need to give bounds on the parameters.

a) (2 points) Parametrize $y = \cos(3x) - \sin(3z)$ as

$$\vec{r}(x, z) =$$

b) (2 points) Parametrize $\rho = 2 + \cos(8\theta + 5\phi)$ as

$$\vec{r}(\theta, \phi) =$$

c) (2 points) Parametrize $r^2 - z^2 = 1$ as

$$\vec{r}(\theta, z) =$$

d) (2 points) Parametrize $x = 0$ as

$$\vec{r}(y, z) =$$

e) Decide whether none, one, or both of the grid curves $u = 1$, $v = 1$ is a circle, if

$$\vec{r}(u, v) = \langle (3u + u \cos(v)) \cos(2u), (3u + u \cos(v)) \sin(2u), (3u + u \sin(v)) \rangle$$

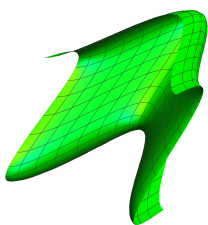
(1 point) Is the curve $u = 1$ a circle?

Yes or No

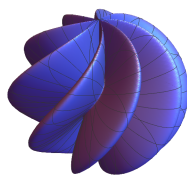
(1 point) Is the curve $v = 1$ a circle?

Yes or No

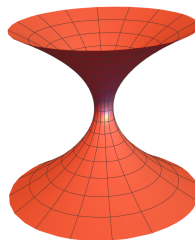
Illustrations:



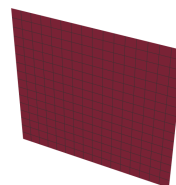
a)



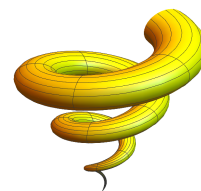
b)



c)



d)



e)

Problem 10) (10 points)

We enjoy the summer and ask in a local restaurant for a refreshment. In each of the ordered items, give a **surface parametrization** of the form

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle .$$

As indicated, we can use also other variables. Your task is to fill in the three parametrization functions in each case, using the variables provided.

a) (2 points) Parametrize the **lemonade glass** $x^2 + y^2 = 1$.

$$\vec{r}(\theta, z) = \langle \boxed{}, \boxed{}, \boxed{} \rangle .$$

b) (2 points) Parametrize the **sorbet glass** $x^2 + y^2 = z^2$.

$$\vec{r}(\theta, z) = \langle \boxed{}, \boxed{}, \boxed{} \rangle .$$

c) (2 points) Parametrize the **lemon** surface $x^2 + y^2 = \sin(z)$.

$$\vec{r}(\theta, z) = \langle \boxed{}, \boxed{}, \boxed{} \rangle .$$

d) (2 points) Parametrize one of the **chips** $z = x^2 - y^2$.

$$\vec{r}(x, y) = \langle \boxed{}, \boxed{}, \boxed{} \rangle .$$

e) (2 points) Parametrize the **lime** $x^2 + y^2 + (z - 3)^2 = 1$.

$$\vec{r}(\theta, \phi) = \langle \boxed{}, \boxed{}, \boxed{} \rangle .$$

