

Name:

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|--------------------------|
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| MWF 12 Jun-Hou Fung |
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| TTH 10 Ziliang Che |
| TTH 10 George Melvin |
| TTH 11:30 Jake Marcinek |
| TTH 11:30 George Melvin |

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

| | | |
|--------|--|-----|
| 1 | | 20 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| 9 | | 10 |
| 10 | | 10 |
| Total: | | 110 |

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.)

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|---|---|
| T | F |
|---|---|

The
(ρ, θ)

- 2)

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| T | F |
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 If $|\vec{v} \times \vec{w}| = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.
- 3)

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 The surface $z^2 + 4y^2 = x^2 + 1$ is a two sheeted hyperboloid.
- 4)

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 The surface $4x^2 - 4x + y^2 - 2y - 120 = -z^2$ is an ellipsoid.
- 5)

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 The parametrized lines $\vec{u}(t) = \langle 1 + 2t, 2 - 5t, 1 + t \rangle$ and $\vec{v}(t) = \langle 3 - 4t, -3 + 10t, 2 - 2t \rangle$ are the same line.
- 6)

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 The surface $\sin(x) = z$ contains lines which are parallel to the y-axis.
- 7)

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 If $\vec{u} \cdot \vec{v} = 0$, $\vec{v} \cdot \vec{w} = 0$ and \vec{v} is not the zero vector, then $\vec{u} \cdot \vec{w} = 0$.
- 8)

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 The curvature of a curve depends upon the speed at which one travels upon it.
- 9)

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| T | F |
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 Two lines in space that do not intersect must be parallel.
- 10)

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| T | F |
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 A line in space can intersect an elliptic paraboloid in 4 points.
- 11)

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| T | F |
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 If $\vec{u} \times \vec{v} = 0$ and $\vec{u} \cdot \vec{v} = 0$, then one of the vectors \vec{u} and \vec{v} is zero.
- 12)

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 If the velocity vector $\vec{r}'(t)$ and the acceleration vector $\vec{r}''(t)$ of a curve are parallel at time $t = 1$, then the curvature $\kappa(t)$ of the curve is zero at time $t = 1$.
- 13)

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 If the speed of a parametrized curve is constant over time, then the curvature of the curve $\vec{r}(t)$ is zero.
- 14)

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| T | F |
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 The length of the vector projection of a vector \vec{v} onto a vector \vec{w} is always equal to the length of the vector projection of \vec{w} onto \vec{v} .
- 15)

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| T | F |
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 A quadric $ax^2 + by^2 + cz^2 = 1$ is contained in the interior of a sphere $x^2 + y^2 + z^2 < 100$, then the constants a, b, c are all positive and the quadric is an ellipsoid.
- 16)

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| T | F |
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 There is a hyperboloid of the form $ax^2 + by^2 - cz^2 = 1$ which has a trace which is a parabola.
- 17)

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| T | F |
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 The set of points in space which have distance 1 from the line $x = y = z$ form a cylinder.
- 18)

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| T | F |
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 The velocity vector of a parametric curve $\vec{r}(t)$ always has constant length.
- 19)

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|---|---|
| T | F |
|---|---|

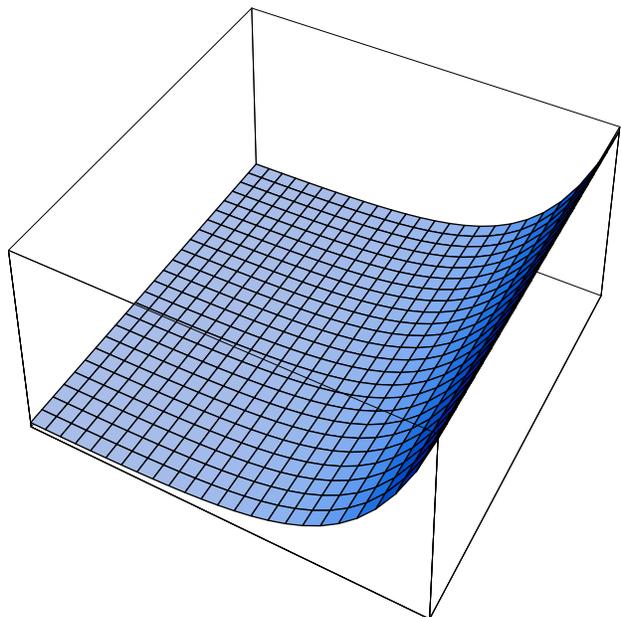
 The volume of a parallelepiped spanned by $\vec{u}, \vec{v}, \vec{w}$ is $|(\vec{u} \times \vec{v}) \cdot \vec{w}|$.
- 20)

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| T | F |
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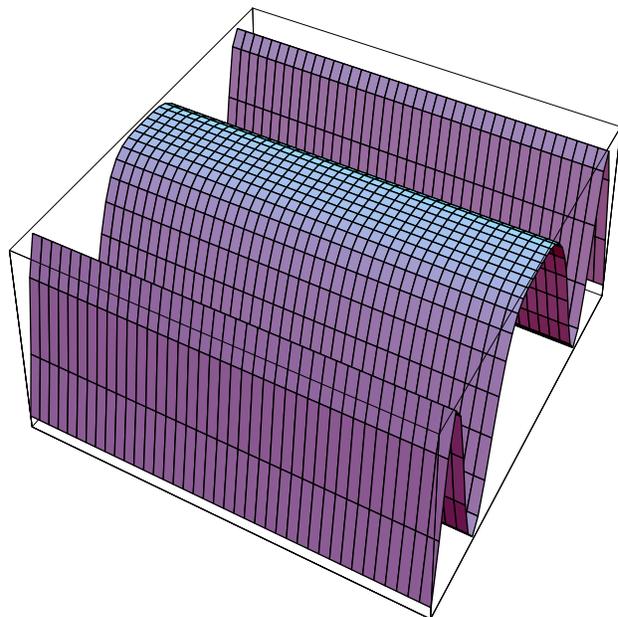
 The equation $x^2 + y^2/4 = 1$ in space describes an ellipsoid.

Problem 2a) (3 points)

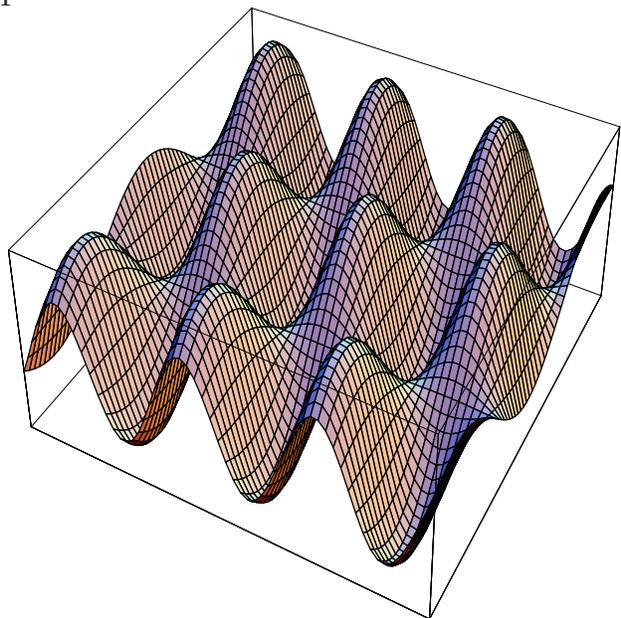
Match the equation with their graphs. No justifications are needed.



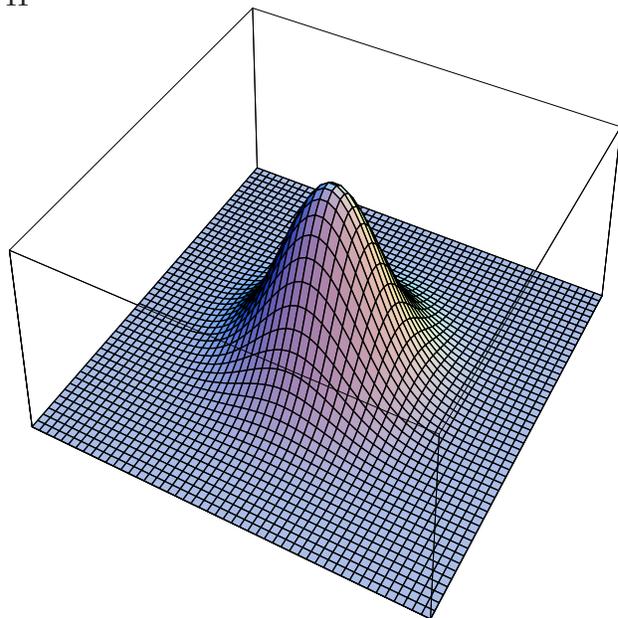
I



II



III

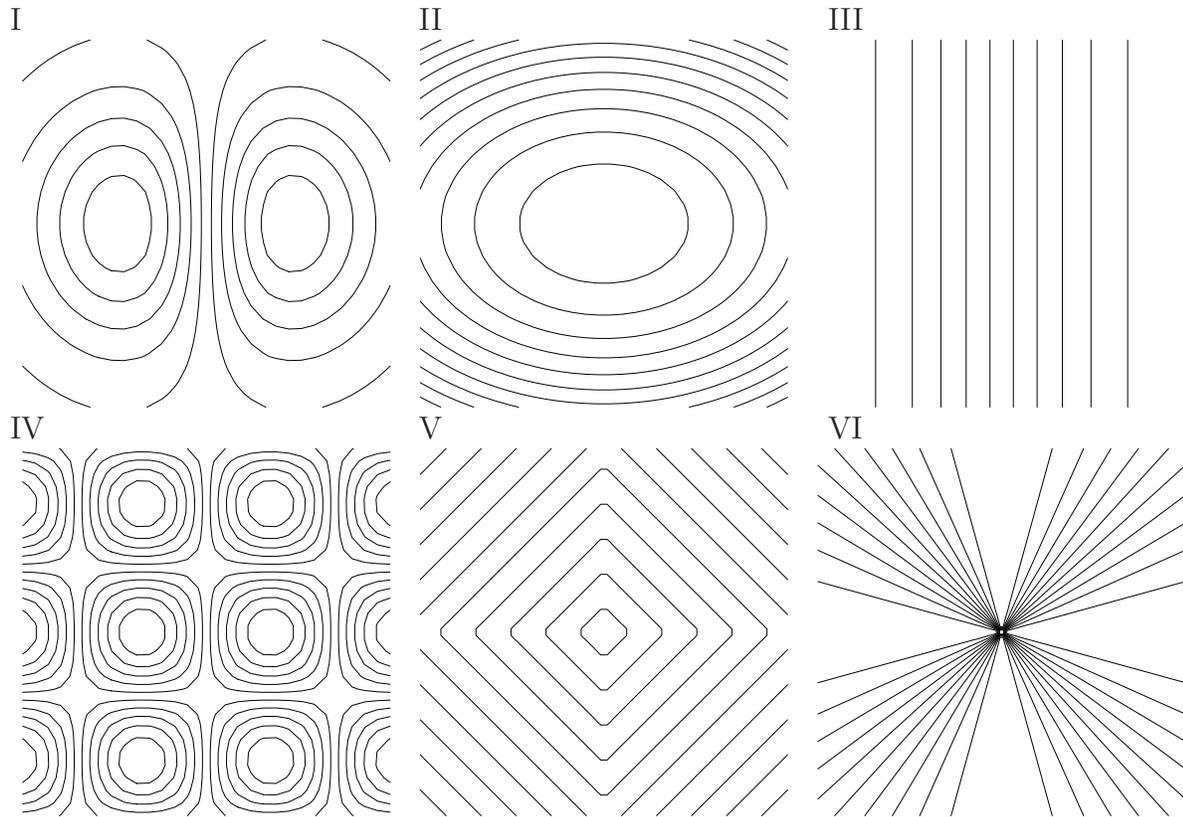


IV

| Enter I,II,III,IV here | Equation |
|------------------------|-------------------------|
| | $z = \sin(5x) \cos(2y)$ |
| | $z = \cos(y^2)$ |
| | $z = e^{-x^2-y^2}$ |
| | $z = e^x$ |

Problem 2b) (4 points)

Match the contour maps with the corresponding functions $f(x, y)$ of two variables. No justifications are needed.



| Enter I,II,III,IV,V or VI here | Function $f(x, y)$ |
|--------------------------------|-----------------------------|
| | $f(x, y) = \sin(x)$ |
| | $f(x, y) = x^2 + 2y^2$ |
| | $f(x, y) = x + y $ |
| | $f(x, y) = \sin(x) \cos(y)$ |
| | $f(x, y) = xe^{-x^2-y^2}$ |
| | $f(x, y) = x^2/(x^2 + y^2)$ |

Match the following points in cartesian coordinates with the points in spherical coordinates:

a) $(x, y, z) = (\sqrt{2}, 0, 0)$

b) $(x, y, z) = (0, \sqrt{2}, 0)$

c) $(x, y, z) = (0, 0, \sqrt{2})$

d) $(x, y, z) = (1, 1, 0)$

e) $(x, y, z) = (1, 0, 1)$

f) $(x, y, z) = (0, 1, 1)$

1) $(\rho, \phi, \theta) = (\sqrt{2}, 0, 0)$.

2) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/4)$.

3) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, 0)$.

4) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/2)$.

5) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, \pi/2)$.

6) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, 0)$.

Problem 3) (10 points)

- a) (7 points) Find a parametric equation for the line which is the intersection of the two planes $2x - y + 3z = 9$ and $x + 2y + 3z = -7$.
- b) (3 points) Find a plane perpendicular to both planes given in a) which has the additional property that it passes through the point $P = (1, 1, 1)$.

Problem 4) (10 points)

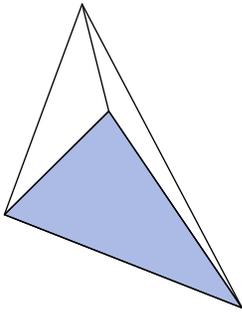
Given the vectors $\vec{v} = \langle 1, 1, 0 \rangle$ and $\vec{w} = \langle 0, 0, 1 \rangle$ and the point $P = (2, 4, -2)$. Let Σ be the plane which goes through the origin $(0, 0, 0)$ and which contains the vectors \vec{v} and \vec{w} . Let S be the unit sphere $x^2 + y^2 + z^2 = 1$.

- a) (6 points) Compute the distance from P to the plane Σ .
- b) (4 points) Find the shortest distance from P to the sphere S .

Problem 5) (10 points)

- a) (6 points) Find an equation for the plane through the points $A = (0, 1, 0)$, $B = (1, 2, 1)$ and $C = (2, 4, 5)$.
- b) (4 points) Given an additional point $P = (-1, 2, 3)$, what is the volume of the tetrahedron which has A, B, C, P among its vertices.

A useful fact which you can use without justification in b): the volume of the tetrahedron is $1/6$ of the volume of the parallelepiped which has $AB, AC,$ and AP among its edges.



Problem 6) (10 points)

The parametrized curve $\vec{u}(t) = \langle t, t^2, t^3 \rangle$ (known as the "twisted cubic") intersects the parametrized line $\vec{v}(s) = \langle 1 + 3s, 1 - s, 1 + 2s \rangle$ at a point P . Find the angle of intersection.

Problem 7) (10 points)

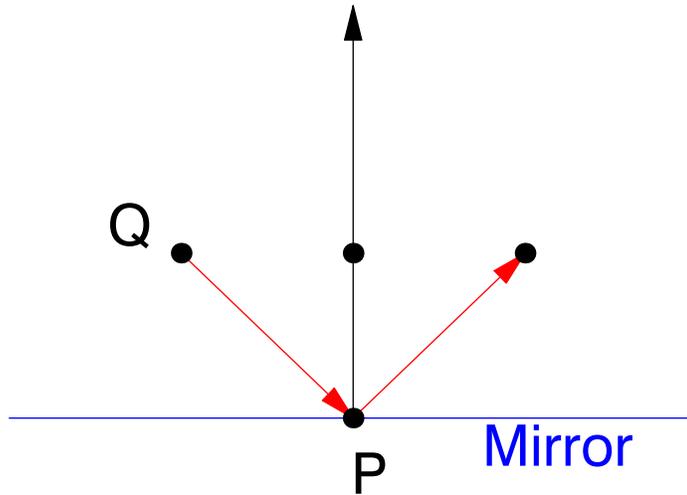
Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (\log(t), 2t, t^2)$, where $\log(t)$ is the natural logarithm (denoted by $\ln(t)$ in some textbooks).

- a) What is the velocity and what is the acceleration at time $t = 1$?
- b) Find the length of the curve from $t = 1$ to $t = 2$.

Problem 8) (10 points)

A planar mirror in space contains the point $P = (4, 1, 5)$ and is perpendicular to the vector $\vec{n} = \langle 1, 2, -3 \rangle$. The light ray $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$ with source $Q = (7, 0, 7)$ hits the mirror plane at the point P .

- a) (4 points) Compute the projection $\vec{u} = \vec{P}_{\vec{n}}(\vec{v})$ of \vec{v} onto \vec{n} .
- b) (6 points) Identify \vec{u} in the figure and use it to find a vector parallel to the reflected ray.



Problem 9) (10 points)

We know the acceleration $\vec{r}''(t) = \langle 2, 1, 3 \rangle + t\langle 1, -1, 1 \rangle$ and the initial position $\vec{r}(0) = \langle 0, 0, 0 \rangle$ and initial velocity $\vec{r}'(0) = \langle 11, 7, 0 \rangle$ of an unknown curve $\vec{r}(t)$. Find $\vec{r}(6)$.

Problem 10) (10 points)

Intersecting the elliptic cylinder $x^2 + y^2/4 = 1$ with the plane $z = \sqrt{3}x$ gives a curve in space.

- (3 points) Find the parametrization of the curve.
- (3 points) Compute the unit tangent vector \vec{T} to the curve at the point $(0, 2, 0)$.
- (4 points) Write down the arc length integral and evaluate the arc length of the curve.