Name:

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TTH 11:30 Jake Marcinek
TTH 11:30 George Melvin

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

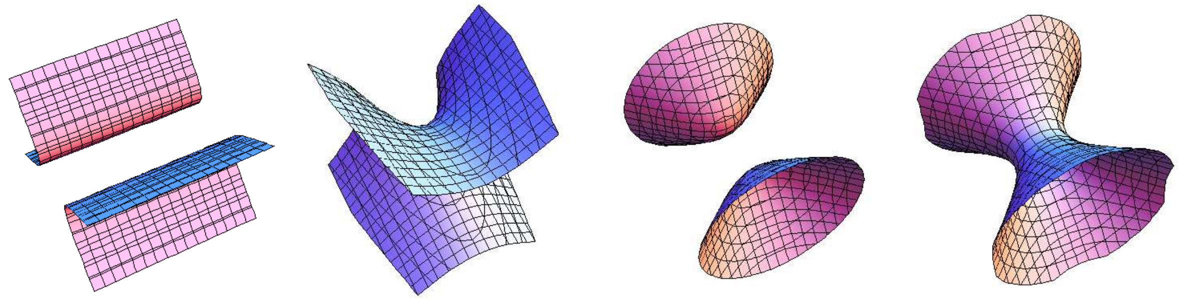
Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed. 1) T F The

2)	T	F	The length of the unit tangent vector $\vec{T}$ for a curve $\vec{r}(t)$ is independent of t.
3)	T	F	For all vectors $\vec{v}$ and $\vec{w}$ the vector $\vec{w} \times (\vec{w} \times \vec{v})$ is perpendicular to $\vec{v}$.
4)	T	F	There is a point (x,y,z) in space, for which the cylindrical coordinates (r,θ,z) and spherical coorindates (ρ,θ,ϕ) satisfy $(r,\theta,z) = (\rho,\theta,\phi - \pi/2)$.
5)	T	F	The two planes $x+y-z=1$ and $-x-y+z=2$ intersect in a line.
6)	T	F	$(\vec{u}+\vec{v}) \cdot (\vec{u}-\vec{v}) = 0$ implies $\|u\| = \|v\|$.
7)	T	F	The contour curves $\sin(x)+y=1$ and $\sin(x)+y=2$ do not intersect.
8)	T	F	There is a vector $\vec{v}$ for which the vector projection $\text{proj}_{\vec{v}}(\vec{j})$ is equal to $2\vec{j}$.
9)	T	F	$(\vec{k} \times \vec{i}) \times \vec{i} = \vec{j} \times (\vec{i} \times \vec{k})$
10)	T	F	If a curve $\vec{r}(t)$ lies in a plane, goes through the point $(1,1,1)$, and has the binormal vector $\vec{B}(t) = \langle 3,4,5 \rangle$, then the plane is $3x+4y+5z=12$.
11)	T	F	The angle between $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ is always 90 degrees.
12)	T	F	A line intersects a hyperbolic paraboloid always in 2 distinct points.
13)	T	F	There is a quadric surface, each of whose intersections with the coordinate planes is either an ellipse or a parabola.
14)	T	F	The equation $x^2 - y^2 - z^2 = 1$ defines a one-sheeted hyperboloid.
15)	T	F	The function $f(x,y) = 1/(1+x^2+y^2)$ is continuous everywhere.
16)	T	F	If the number $\vec{u} \cdot (\vec{v} \times \vec{w})$ is positive, then $(\vec{w} \times \vec{v}) \cdot \vec{u}$ is positive.
17)	T	F	The number $\|\vec{u} \times (\vec{v} \times \vec{w})\|$ is the volume of the parallelepiped spanned by $\vec{u}, \vec{v}$ and $\vec{w}$.
18)	T	F	The set of points P for which the distance of P to the point $(0,0,0)$ is 1 less than the distance to the point $(0,0,2)$ is a paraboloid.
19)	T	F	If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $\text{proj}_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$.
20)	T	F	The number $\|\vec{v} \times \vec{w}\|$ is the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$.

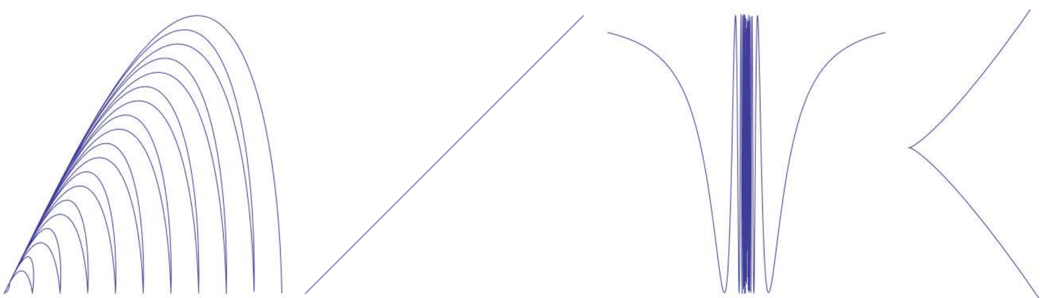
Problem 2a) (5 points)

Match the equations with the pictures. No justifications are necessary in this problem.



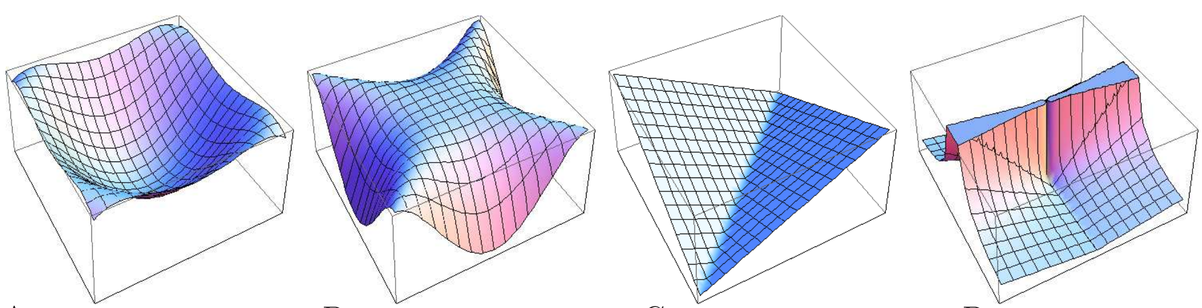
I II III IV

Enter I,II,III,IV here	Equation
	$x^2 + y - z^2 - 1 = 0$
	$y^2 - 2z^2 - 1 = 0$
	$x^2 - y^2 + z^2 + 1 = 0$
	$x^2 - y^2 + z^2 - 1 = 0$



1 2 3 4

Enter 1,2,3,4 here	Equation
	$\langle t^4, 1 + t^5 \rangle$
	$\langle t\cos(5t), t\cos(5t) \rangle$
	$\langle \lvert t\cos(5t) \rvert, \lvert t\sin(10t) \rvert \rangle$
	$\langle 3 + 2t, \cos(1/t) \rangle$

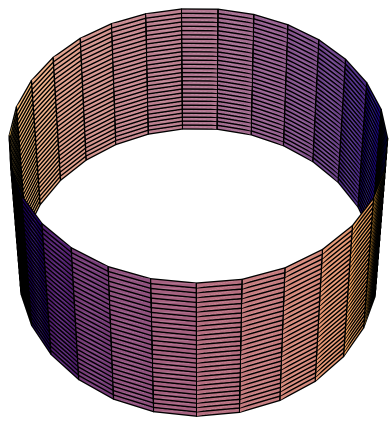


A B C D

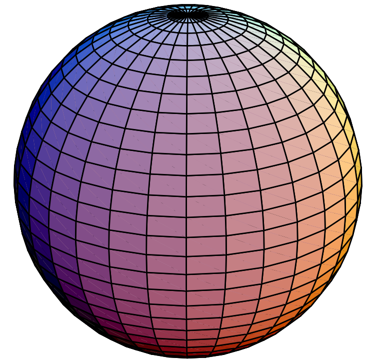
Enter A,B,C,D here	Equation
	$f(x, y) = \lvert x/y \rvert$
	$f(x, y) = \sin(x^2 + y^2)$
	$f(x, y) = \lvert x - y \rvert$
	$f(x, y) = \cos(x^2 - y^2)$

Problem 2b) (5 points)

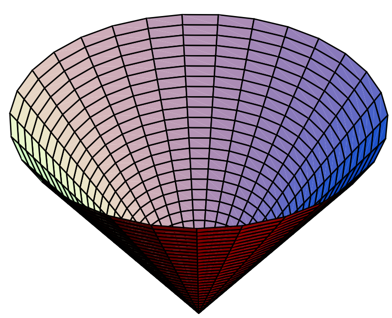
Match the surfaces with their parametrizations as well as with the description either in cylindrical coordinates (r, θ, z) or in spherical coordinates (ρ, ϕ, θ).



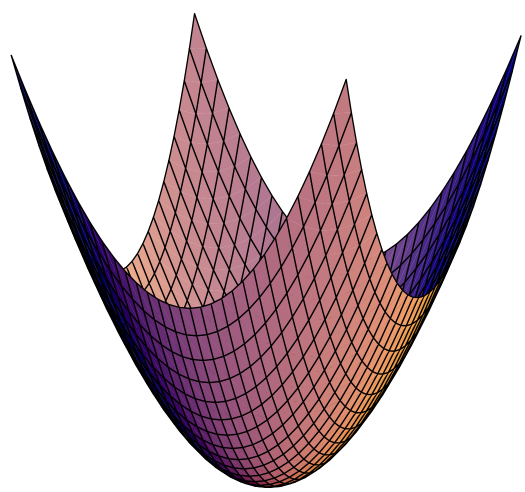
I



II



III



IV

Enter I,II,III,IV here	Parametrization of the surface
	$\langle 3\cos(\theta), 3\sin(\theta), 2z \rangle$.
	$\langle x, y, 3x^2 + 3y^2 \rangle$
	$\langle 3\sin(\phi)\cos(\theta), 3\sin(\phi)\sin(\theta), 3\cos(\phi) \rangle$
	$\langle 3z\cos(\theta), 3z\sin(\theta), 2z \rangle$

Enter I,II,III,IV here	Description in cylindrical or spherical coordinates
	$z = 3r^2$
	$3z = 2r$
	$\rho = 3$
	$r = 3$

Problem 3) (10 points)

A tetrahedron has the vertices $A = (1, 1, 0), B = (3, 2, 0), C = (2, 1, 1), D = (3, 2, 1)$ with base triangle A, B, C.

a) (5 points) Find the height of the tetrahedron.

b) (5 points) The volume of a tetrahedron is the base area times height divided by 3. What is the volume of the tetrahedron with vertices A,B,C,D.

Problem 4) (10 points)

Find a parametrization of the line containing the two planes

$$2x + y + z = 4$$

and

$$x - y + 2z = 5 \ .$$

Problem 5) (10 points)

What is the distance between the two cylinders $x^2 + y^2 = 1$ and $(z - 2)^2 + (x - 5)^2 = 4$?

Problem 6) (10 points)

Find the arc length of the parameterized curve

$$\vec{r}(t) = \langle 2\sin(t), \frac{t^4}{4} + \frac{1}{2t^2}, 2\cos(t)\rangle$$

from $t = 1$ to $t = 2$.

Problem 7) (10 points)

At time $t = 0$ two trapeze artists have positions $\vec{r}(0) = \langle 0, 0, 25 \rangle$ and $\vec{s}(0) = \langle 10, 0, 23 \rangle$ and velocities $\vec{r}\,'(0) = \langle 2, 0, 1 \rangle$ and $\vec{s}\,'(0) = \langle -3, 0, 2 \rangle$. They both experience a constant gravitational acceleration $\langle 0, 0, -10 \rangle$. Find the paths $\vec{r}(t), \vec{s}(t)$ and determine at which point the artists meet.

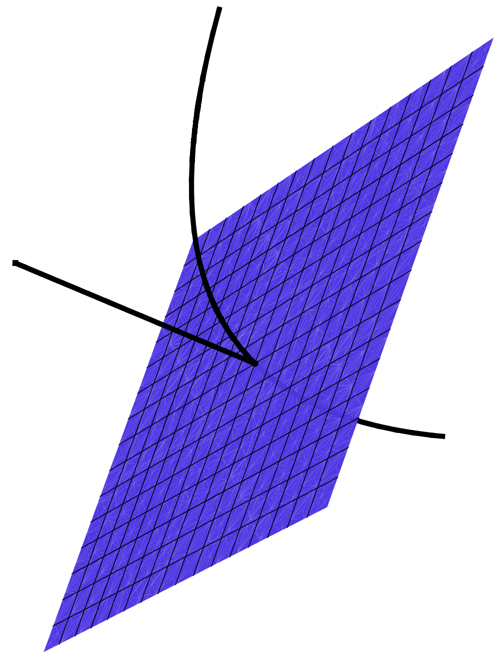
Problem 8) (10 points)

The angle between a curve and a plane is defined as $\pi/2 - \alpha$, where α is the angle between the normal vector to the plane and the velocity vector of the curve at the point of intersection.

a) (3 points) Find a normal vector to the plane $x + y - z/2 = 1$.

b) (3 points) What is the velocity vector to the curve $C : \vec{r}(t) = \langle 1, 0, 0 \rangle + t\langle 1, 1, 3 \rangle + t^2\langle 1, 1, 1 \rangle$ at time $t = 0$?

c) (4 points) Find the angle (in radians) between the plane $x + y - z/2 = 1$ and the curve C at the point of intersection $\vec{r}(0)$.



Problem 9) (10 points)

The intersection of the paraboloid

$$x^2 + y^2 - z = 5$$

with the plane

$$x + y = 5$$

is a curve. Find the parametrization of this curve.

Problem 10) (10 points)

a) (3 points) Parametrize the plane containing the three points $A = (1, 1, 1), B = (1, 3, 2)$ and $C = (3, 4, 5)$.
b) (4 points) Parametrize the sphere which is centered at $(1, 1, 1)$ and has radius 3.
c) (3 points) Parametrize the surface which is given in spherical coordinates as $\rho = 3 + \sin(\phi)\sin(\theta)$.