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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3,8, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points), no justifications needed

- 1)

T	F
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 There is a function $f(x, y)$ for which the linearization at $(0, 0)$ is $L(x, y) = x^2 + y^2$.
- 2)

T	F
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 For any two functions f, g and unit vector $\vec{u}$ we have $D_{\vec{u}}(f + g) = D_{\vec{u}}f + D_{\vec{u}}g$.
- 3)

T	F
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 $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx = \int_0^2 \int_0^{\pi/2} r^2 d\theta dr$.
- 4)

T	F
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 If we solve $\sin(y) - xy^2 = 0$ for y , then $y' = -y^2/(\cos(y) - 2xy)$.
- 5)

T	F
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 If $f(x, 0) = 0$ for all x and $f(0, y) = 0$ for all y , then $g(x, y) = \int_0^x \int_0^y f(s, t) dt ds$ solves $g_{xy}(x, y) = f(x, y)$.
- 6)

T	F
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 If $|\nabla f| = 1$ at $(0, 0)$, then there exists a direction in which the slope of the graph of f at $(0, 0)$ is 1.
- 7)

T	F
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 The function $f(x, y) = x^2 + y^2$ satisfies the partial differential equation $f_{xx}f_{yy} - f_{xy}^2 = 4$.
- 8)

T	F
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 The height of Mount Wachusett is $f(x, y) = 4 - 2x^2 - y^2$. On the trail $x^2 + y^2 = 1$, the point $(1, 0)$ is a maximum.
- 9)

T	F
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 Mount Wachusett has height $f(x, y) = 4 - 2x^2 - y^2$. Except at the maximum $(0, 0)$, the gradient vector is perpendicular to the graph of the function.
- 10)

T	F
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 If $f_x(a, b) > 0$ and $f_y(a, b) > 0$ then for any unit vector $\vec{u}$ we must have $D_{\vec{u}}f(a, b) > 0$.
- 11)

T	F
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 If $f(x, y)$ has two local minima, then f must have at least one local maximum.
- 12)

T	F
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 If $\vec{r}(t)$ is a curve on the surface $g(x, y, z) = x^2 + y^2 - z^2 = 6$ then $\nabla g(\vec{r}(t)) \cdot \vec{r}'(t) = 0$.
- 13)

T	F
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 If f and g have the same trace $\{x = 5\}$ then $f_x(5, y) = g_x(5, y)$ for all y .
- 14)

T	F
---	---

 If f and g have the same trace $\{x = 5\}$ then $f_y(5, y) = g_y(5, y)$ for all y .
- 15)

T	F
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 The surface area of $\vec{r}_1(u, v) = \langle u \cos(v), u \sin(v), u^2 \rangle$ and $\vec{r}_2(u, v) = \langle \sqrt{u} \cos(v), \sqrt{u} \sin(v), u \rangle$ defined on $\{0 \leq u, v \leq 1\}$ are the same.
- 16)

T	F
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 If $\vec{r}(t)$ is a curve on a graph $z = f(x, y)$ of a function $f(x, y)$, then the velocity vector of $\vec{r}$ is perpendicular to the vector $\langle f_x, f_y, -1 \rangle$.
- 17)

T	F
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 A continuous function $f(x, y)$ on the closed disc $R = \{x^2 + y^2 \leq 51^2\}$ (of course, R is called “**area** 51π ”) has a global maximum on R .
- 18)

T	F
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 Any continuous function $f(x, y)$ has a global minimum and maximum on the curve $y = x^2$.
- 19)

T	F
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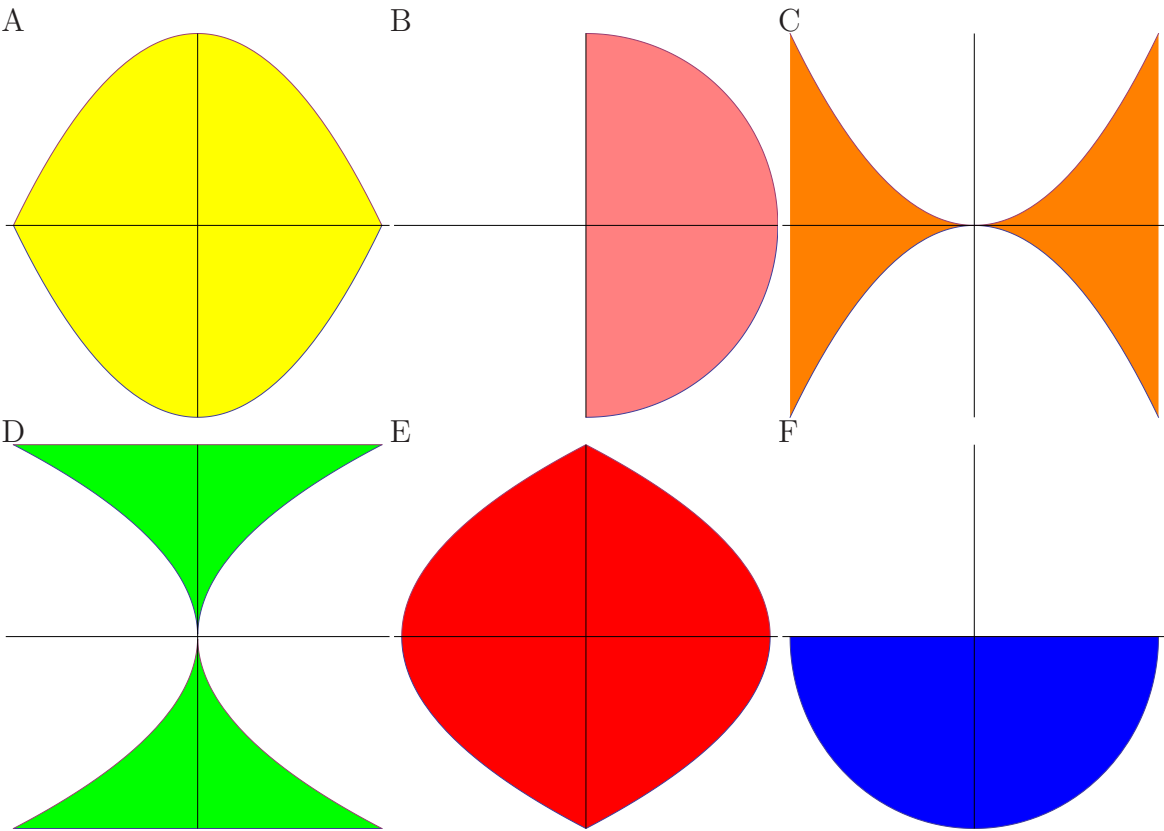
 Fubini's theorem assures that $\int_a^b \int_c^d f(x, y) dy dx = \int_a^b \int_c^d f(x, y) dx dy$.
- 20)

T	F
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 $\int \int_R \sin(x + y) dx dy = 0$ for $R = \{-\pi \leq x \leq \pi, -\pi \leq y \leq \pi\}$.

Problem 2) (10 points)

a) (6 points) Match the integration regions with the integrals. Each integral matches exactly one region $A - F$.



Enter A-F	Integral
	$\int_{-1}^1 \int_{-x^2}^{x^2} f(x, y) \, dydx.$
	$\int_{-1}^1 \int_{-y^2}^{y^2} f(x, y) \, dxdy.$
	$\int_{-1}^1 \int_{y^2-1}^{1-y^2} f(x, y) \, dxdy.$
	$\int_{-1}^1 \int_0^{\sqrt{1-y^2}} f(x, y) \, dxdy.$
	$\int_{-1}^1 \int_{x^2-1}^{1-x^2} f(x, y) \, dydx.$
	$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^0 f(x, y) \, dydx.$

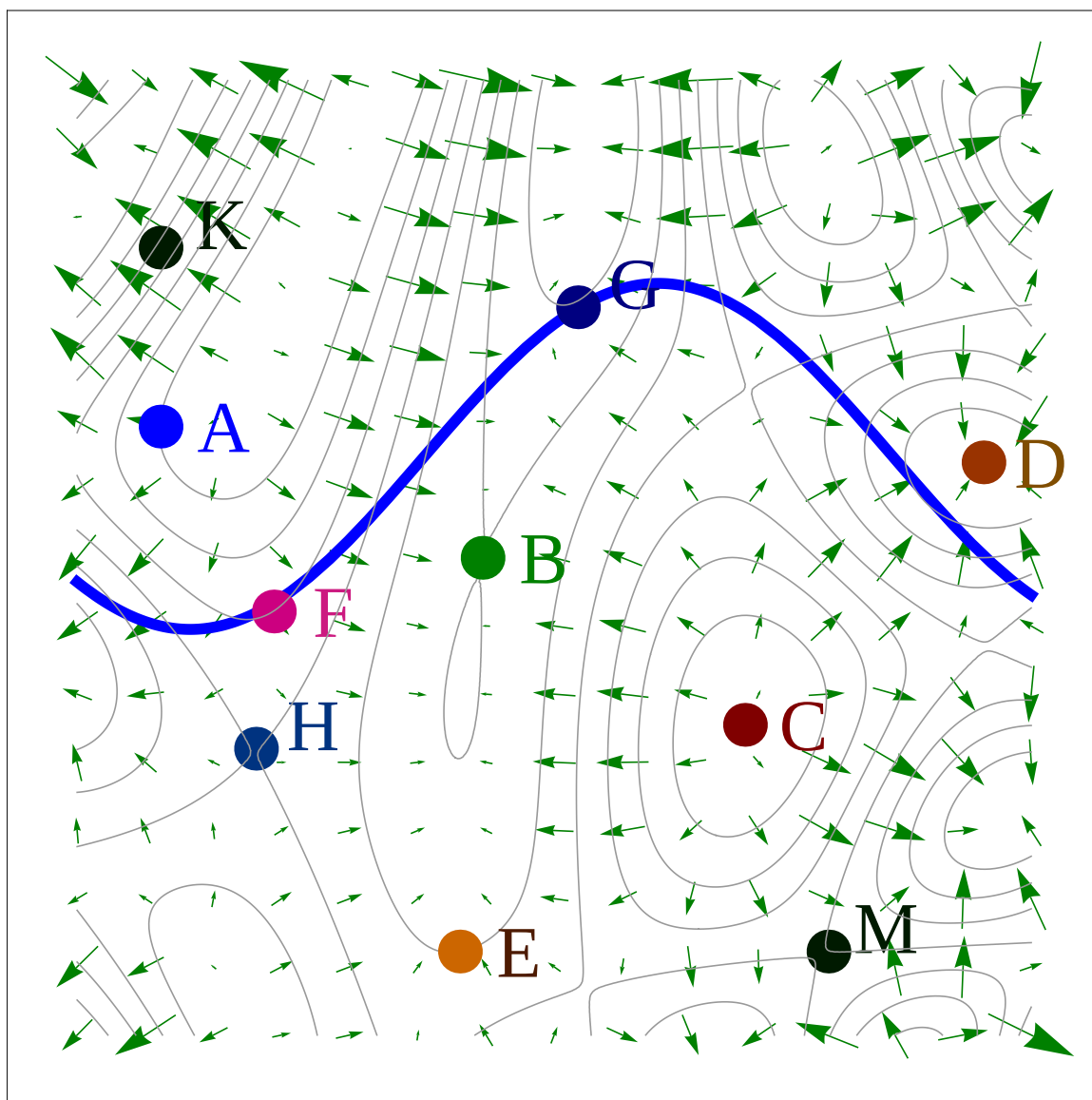
b) (4 points) Fill in one word names (like “Heat”, “Wave” etc) for the partial differential equations:

Enter one word	PDE
	$g_x = g_y$
	$g_{xx} = g_{yy}$
	$g_{xx} = -g_{yy}$
	$g_x = g_{yy}$

Problem 3) (10 points)

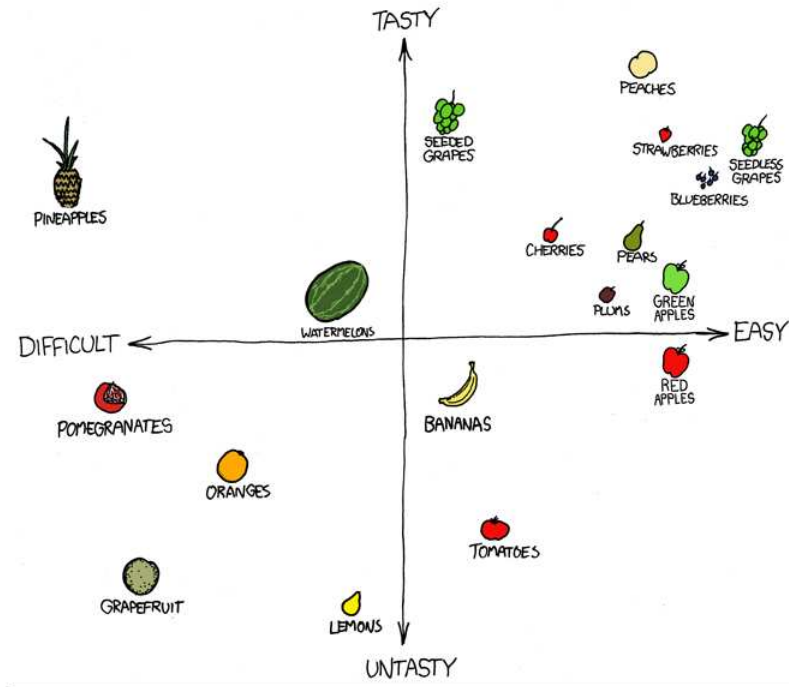
(10 points) A function $f(x, y)$ of two variables has level curves as shown in the picture. We also see a constraint in the form of a curve $g(x, y) = 0$ which has the shape of the graph of the cos function. The arrows show the gradient. In this problem, each of the 10 letters $A, B, C, D, E, F, G, H, K, M$ appears exactly once.

Enter A-P	Description
	a local maximum of $f(x, y)$.
	a local minimum of $f(x, y)$.
	a saddle point of $f(x, y)$ where $f_{xx} < 0$.
	a saddle point of $f(x, y)$ where $f_{xx} > 0$.
	a saddle point of $f(x, y)$ where f_{xx} is close to zero
	a point, where $f_x = 0$ and $f_y \neq 0$
	a point, where $f_y = 0$ and $f_x \neq 0$
	the point, where $ \nabla f $ is largest
	a local maximum of $f(x, y)$ under the constraint $g(x, y) = 0$.
	a local minimum of $f(x, y)$ under the constraint $g(x, y) = 0$.



Problem 4) (10 points)

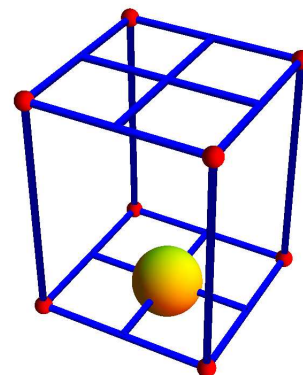
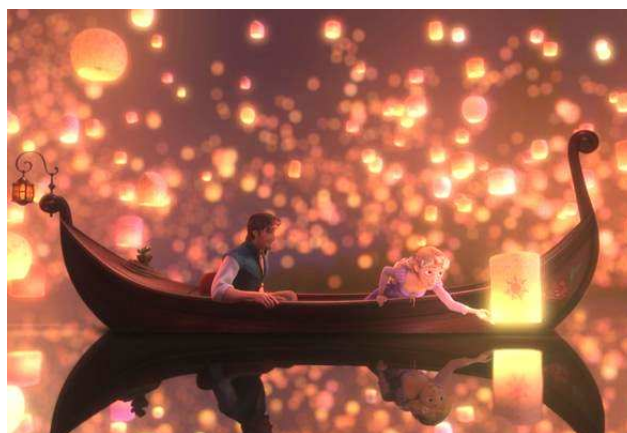
Find and classify all the extrema of the function $f(x, y) = x^5 + y^3 - 5x - 3y$. This function measures “eat temptation” in the x =Easy- y =Tasty plane. Is there a global minimum or global maximum?



The “Easy-Tasty plane” was introduced in the XKCD cartoon titled “F&#% Grapefruits”.

Problem 5) (10 points)

After having watched the latest Disney movie “Tangled”, we want to build a hot air balloon with a cuboid mesh of dimension x, y, z which together with the top and bottom fortifications uses wires of total length $g(x, y, z) = 6x + 6y + 4z = 32$. Find the balloon with maximal volume $f(x, y, z) = xyz$.



Problem 6) (10 points)

- a) (8 points) Find the tangent plane to the surface $f(x, y, z) = x^2 - y^2 + z = 6$ at the point $(2, 1, 3)$.
- b) (2 points) A curve $\vec{r}(t)$ on that tangent plane of the function $f(x, y, z)$ in a) has constant speed $|\vec{r}'| = 1$ and passes through the point $(2, 1, 3)$ at $t = 0$. What is $\frac{d}{dt}f(\vec{r}(t))$ at $t = 0$?

Problem 7) (10 points)

- a) (5 points) Estimate $\sqrt{\sin(0.0004) + 1.001^2}$ using linear approximation.
- b) (5 points) We know $f(0, 0) = 1$, $D_{\langle \frac{3}{5}, \frac{4}{5} \rangle} f(0, 0) = 2$ and $D_{\langle -\frac{4}{5}, \frac{3}{5} \rangle} f(0, 0) = -1$. If $L(x, y)$ is the linear approximation to $f(x, y)$ at the point $(0, 0)$, find $L(0.06, 0.08)$.

Problem 8) (10 points)

- a) (5 points) Find the following double integral

$$\int_0^1 \int_{x^2}^{\sqrt{x}} \frac{\pi \sin(\pi y)}{y^2 - \sqrt{y}} dy dx .$$

- b) (5 points) Evaluate the following double integral

$$\int \int_R \frac{\sin(\pi \sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} dx dy$$

over the region

$$R = \{x^2 + y^2 \leq 1, x > 0\} .$$

Problem 9) (10 points)

- a) (8 points) Find the surface area of the surface parametrized as

$$\vec{r}(u, v) = \langle u - v, u + v, (u^2 - v^2)/2 \rangle ,$$

where (u, v) is in the unit disc $R = \{u^2 + v^2 \leq 1\}$.

- b) (2 points) Give a nonzero vector $\vec{n}$ normal to the surface at $\vec{r}(4, 2) = \langle 2, 6, 6 \rangle$.

Problem 10) (10 points)

a) (6 points) Integrate

$$\int_0^{\pi/2} \int_x^{\pi/2} \frac{\cos(y)}{y} dy dx$$

b) (4 points) Find the moment of inertia

$$\iint_R (x^2 + y^2) dy dx ,$$

where R is the ring $1 \leq x^2 + y^2 \leq 9$.