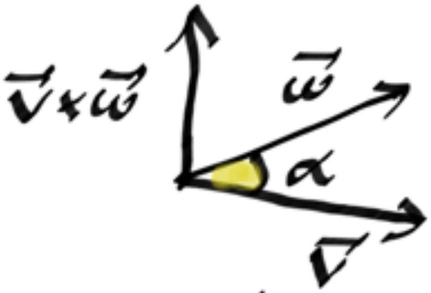
$|\vec v|^2 = \vec v \cdot \vec v$

$\vec v \cdot \vec w = |\vec v| \cdot |\vec w| \cdot \cos\alpha$

angle



$\vec v \cdot (\vec v \times \vec w) = 0$

$\vec w \cdot (\vec v \times \vec w) = 0$

$|\vec v \times \vec w| = |\vec v| |\vec w| \cdot \sin\alpha$

area

$\vec P = \dfrac{\vec v \cdot \vec w}{|\vec w|^2} \vec w$ projection

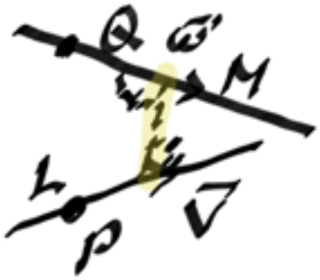
$|\vec u \cdot \vec v \times \vec w|$ volume

$d(P,Q) = |\vec{PQ}|$ distance

$d(P,L) = \dfrac{|\vec{QP} \times \vec v|}{|\vec v|}$

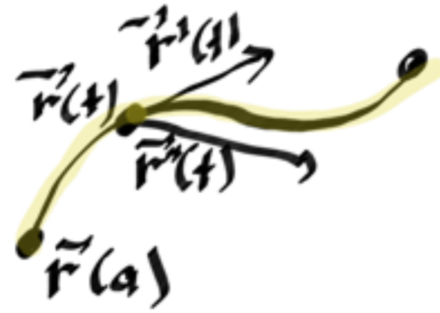
= area / base

$d(P,\Sigma) = \dfrac{\vec{PQ} \cdot \vec n}{|\vec n|}$



$d(L,M) = \dfrac{|\vec{PQ} \cdot (\vec v \times \vec w)|}{|\vec v \times \vec w|}$

$= \dfrac{\text{volume}}{\text{area}}$



parametrized curve.

velocity

$\vec{r}' = \langle x'(t), y'(t), z'(t) \rangle$

$\vec{r}'' =$ acceleration

$|\vec{r}'| =$ speed

$\vec{T} = \dfrac{\vec{r}'}{|\vec{r}'|}$

$\vec{N} = \dfrac{\vec{T}'}{|\vec{T}'|}$

TNB frame

$\vec{B} = \vec{T} \times \vec{N}$

$\kappa = \dfrac{|\vec{T}'|}{|\vec{r}'|} = \dfrac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3}$ curvature

$\vec{r}'(t) = \int_0^t \vec{r}''(s)\, ds$, $\vec{r}(t) = \int_0^t \vec{r}'(s)\, ds$

$\int_a^b |\vec{r}'(t)|\, dt$ arc length

$\vec{r}(t) = \langle \cos t, \sin t \rangle$

$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

$\vec{r}(t) = \langle t, \sin t \rangle$

$\vec{r}(t) = \langle 1-t, t \rangle$

$\vec{r}(t) = \langle t \cos t, t \sin t \rangle$

$f(x,y,z) = d$ implicit surface

$x^2+y^2=1$ cylinder

$ax+by+cz=d$ plane

$z - x^2 + y^2 = 0$ hyperbolic paraboloid

$x^2+y^2+z^2=1$ sphere

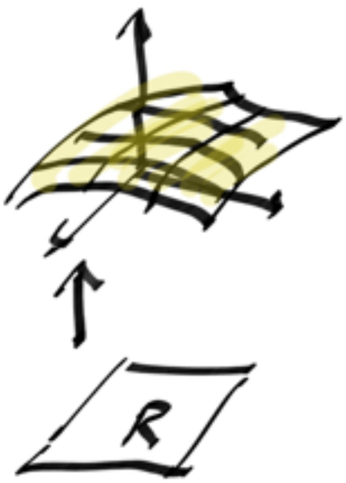
$z - x^2 - y^2 = 0$ paraboloid

$z^2 - x^2 - y^2 = 0$ cone

$z - f(x,y) = 0$ graph

$z^2 - x^2 - y^2 = 1$ two sheet hyperb.

$z^2 - x^2 - y^2 = -1$ one sheet hyperb.



$\vec F(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

parametrized surface.

$\vec F(u_0,t)$ grid curves $\vec F(t,v_0)$

$\vec F_u \times \vec F_v$ normal vector.

$\iint_R |\vec r_u \times \vec r_v| \, du\,dv$ surface area

$\vec F(\theta,\varphi) = \langle \cos\theta \sin\varphi, \sin\theta \sin\varphi, \cos\varphi \rangle$

$\vec F(z,\theta) = \langle g(z)\cos\theta, g(z)\sin\theta, z \rangle$

$\vec F(x,y) = \langle x, y, f(x,y) \rangle$

$\vec F(s,t) = \vec{OP} + s \cdot \vec v + t \vec w$

sphere, surface of revolution, graphs and planes

$\vec{F} = \langle P, Q \rangle$ curl$(\vec{F}) = Q_x - P_y$

$f(x,y)$ grad$(f) = \langle f_x, f_y \rangle$

$\vec{F} = \langle P, Q, R \rangle$ curl$(F) = \nabla \times \vec{F}$

$= \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$

$f(x,y,z)$ grad$(f) = \langle f_x, f_y, f_z \rangle$

div$(\vec{F}) = P_x + Q_y + R_z$

curl(grad f) $= 0$

div(curl$(\vec{F})$) $= \vec{0}$

$\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$ line integral

$\iint_R \vec{F}(\vec{r}(u,v)) \cdot \vec{r}_u \times \vec{r}_v \;du\,dv$

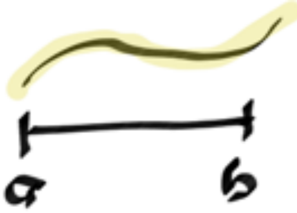
flux integral

$\iint_G f(x,y)\,dx\,dy$ double integral

$\iiint_E f(x,y,z)\,dx\,dy\,dz$ triple integral

$\iint_G 1\,dx\,dy$ area

$\iiint_E 1\,dx\,dy\,dz$ Volume



$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Fundamental theorem of calculus

$$\int_a^b \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt = f(\vec{r}(b)) - f(\vec{r}(a))$$

Fundamental theorem of line integrals

$$\iint_G \text{curl}(\vec{F})\,dx\,dy = \int F(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

greens theorem

$$\int_a^b \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt = f(B) - f(A)$$

Fundamental theorem of line integrals

$$\iint_S \text{curl}\,\vec{F}(\vec{r}(u,v)) \cdot \vec{r}_u \times \vec{r}_v\,du\,dv$$

$$= \int_C F(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

Stokes theorem

$$\iiint_E \text{div}(\vec{F})\,dx\,dy\,dz$$

$$= \iint_S \vec{F}(\vec{r}(u,v)) \cdot \vec{r}_u \times \vec{r}_v\,du\,dv$$

Divergence theorem

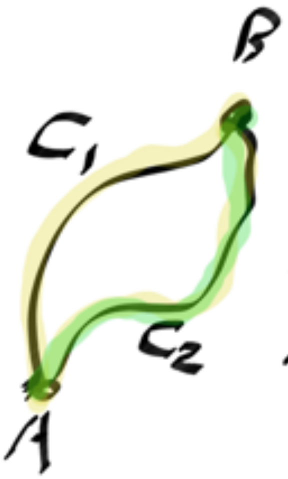
S

closed surface

$$\iint_S \operatorname{curl}(\vec{F})\cdot d\vec{S} = 0$$

closed curve

$$\int_C \nabla f(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt = 0$$



For a field $\vec{F} = \nabla f$ the line integral is path independent

For a field $\vec{F} = \operatorname{curl}(\vec{G})$ the flux integral is surface independent

$\nabla f = \langle a, b, c \rangle$
perpendicular
to $f = c$

$ax + by + cz = d$ tangent plane

$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$
chain rule

$D_{\vec{v}} f = \nabla f \cdot \vec{v}$ directional derivative

$f(x,y,g(x,y)) = 0$

$$g_x = \frac{-f_x}{f_z}, \quad g_y = \frac{-f_y}{f_z}$$

implicit differentiation

$L(x,y) = f(x_0,y_0) + a \cdot (x - x_0) + b\,(y - y_0)$

$\langle a, b \rangle = \nabla f$ linearization
use for estimation

$|\nabla f| =$ maximal steepness

$u_t = u_x$ transport equation

$u_t = u_{xx}$ heat equation

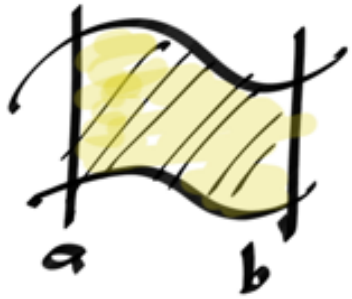
$u_{tt} = u_{xx}$ wave equation

$u_{xx} + u_{yy} = 0$ Laplace equation

$u_t + u u_x = 0$ Burger's equation

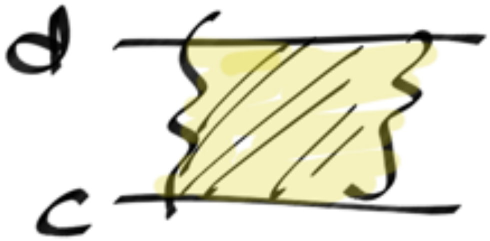
$$\int_a^b \int_c^d f = \int_c^d \int_a^b f$$

Fubini



$$\int_a^b \int_{c(x)}^{d(x)} f(x,y)\,dy\,dx$$

Type I



$$\int_c^d \int_{a(y)}^{b(y)} f(x,y)\,dx\,dy$$

Type II

$$\iint_R \int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\,dx\,dy$$

$$\int_0^{2\pi} \iint_R f(x,y,z)\; r\,dr\,dz\,d\theta$$

$$\int_0^{2\pi} \int_0^{\pi} \int_{g(\theta,\varphi)}^{h(\theta,\varphi)} f(\theta,\varphi,\rho)\;\rho^2 \sin\varphi\,d\rho\,d\theta\,d\varphi$$