

Homework 1: Geometry and Distance

This homework is due Friday, 9/9 respectively Tuesday 9/13 at the beginning of class.

- 1 a) Find the distance a from $P = (2, -5, -7)$ to the xz -plane.
b) Find the distance b from P to the y -axis.
c) Find the distance c from P to the origin $O = (0, 0, 0)$.
d) What is $a^2 + b^2 - c^2$?

- 2 a) Find its center and radius of the sphere S :

$$-x^2 - y^2 - z^2 + 10x + 8y + 2z + 7 = 0 .$$

- b) Find the distance from the center of S to the sphere $x^2 + y^2 + z^2 = 400$.
c) Find the minimal distance between the spheres. This is the minimal distance between two points where each is in one sphere.
- 3 a) Find an equation of the largest sphere with center $(4, 11, 9)$ that is contained in the first octant $\{x \geq 0, y \geq 0, z \geq 0\}$.
b) Find the equation for the sphere centered at $(6, 10, 8)$ which passes through the center $(4, 11, 9)$ of the sphere in a).
- 4 a) What is the surface $(x-1)^2 + z^2 + 4z = 12$ in three dimensional space R^3 .
b) What is the surface $x^2 = z^2$ in three dimensional space R^3 .
c) What is the intersection of the two surfaces? Draw both!

- 5 An ant moves on the unit cube bound by the walls $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ from the point $A = (0.4, 0.8, 1)$ to the point $B = (1, 0.8, 0.5)$. Compute the length of the two obvious paths, where one passes over three faces, the other only over two. Which one is shorter? See the figures on the third page.

Main definitions

A point in the **plane** has **coordinates** $P = (x, y)$. A point in **space** has coordinates $P = (x, y, z)$. The coordinate signs define 4 **quadrants** in the plane and 8 **octants** in space. These regions intersect at the **origin** $O = (0, 0)$ or $O = (0, 0, 0)$, and are separated by **coordinate planes** $\{x = 0\}$, $\{y = 0\}$, $\{z = 0\}$ which intersect in **coordinate axes** like the z -axis $\{y = 0, x = 0\}$.

The **Euclidean distance** between two points $P = (x, y, z)$ and $Q = (a, b, c)$ in space is defined as $d(P, Q) = \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2}$. The distance between a point P and a geometric object S like a line or plane or sphere is the minimal distance $d(P, Q)$ which can be achieved among all points Q located on S .

A **circle** of radius r centered at $P = (a, b)$ is the collection of points in the plane which have distance r from P . A **sphere** of radius ρ centered at $P = (a, b, c)$ is the collection of points in space which have distance ρ from P . The equation of a sphere is $(x - a)^2 + (y - b)^2 + (z - c)^2 = \rho^2$.

We **complete the square** of $x^2 + bx + c = 0$ by adding $(b/2)^2 - c$ on both sides to get $(x + b/2)^2 = (b/2)^2 - c$. Solving for x gives $x = -b/2 \pm \sqrt{(b/2)^2 - c}$. **Example:** Find the center and radius of the circle $x^2 + 8x + y^2 = 9$. **Solution:** Add 16 on both sides to get $x^2 + 8x + 16 + y^2 = 25$ which is $(x + 4)^2 + y^2 = 25$, a circle of radius $r = 5$ centered at $(-4, 0)$.

