Homework 15: Directional Derivatives

This homework is due Monday, 10/17 rsp Tuesday 10/18.

1 Find the gradient of

$$f(x, y, z) = \sqrt{x + yz}\ .$$

at the point $P = (1, 3, 1)$ and use it to find the rate of change of f at P in the direction of the vector $\vec{u} = \langle 2/7, 3/7, 6/7 \rangle$.

2 a) (5 points) Find the directional derivative of the function $f(x, y) = \log(x^2 + y^2)$ at the point $P = (2, 1)$ in the direction of the vector $\vec{v} = \langle -1, 2 \rangle$. (We use the notation $\log = \ln$).
b) (5 points) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at the point $P = (1, -1, 3)$ in the direction from P to $Q = (2, 4, 5)$.

3 a) Find the direction of steepest descent for $f(x, y, z) = \frac{(x+y)}{z}$ at the point $P = (1, 1, -1)$.
b) Find the value of the maximal rate of change at $(1, 1, -1)$ in that direction found in a). This is the directional derivative in that direction.

4 Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

5 On http://goo.gl/Py4id9, you find a map of a neighborhood of Concord MA.
a) The map contains some creeks. Find an example which confirms the rule that water crosses level curves perpendicularly.
b) The map shows also some railway tracks (near Walden pond for example). What is the general relation between level curves

and railway tracks?

c) Estimate the maximal slope on Punkatassett hill, north of Concord.

Main definition:

If f is a function of several variables and $\vec{v}$ is a unit vector then $D_{\vec{v}}f = \nabla f \cdot \vec{v}$ is the **directional derivative** of f in the direction $\vec{v}$.

For $\vec{v} = \nabla f/|\nabla f|$, the directional derivative is

$$D_v f = \nabla f \cdot \nabla f/|\nabla f| = |\nabla f| \,,$$

so that f **increases** in the direction of the gradient. The value $|\nabla f|$ is the maximal slope.