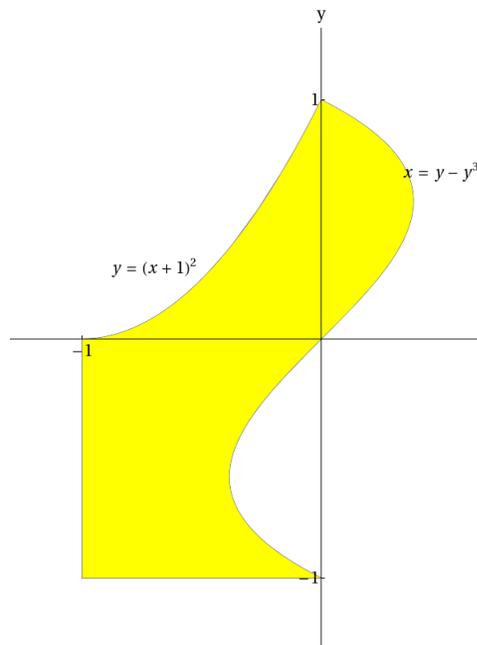


Homework 20: Polar integration

This homework is due Friday, 10/28 rsp Tuesday 11/1.

- 1 This is a review problem from the last section which does not deal with polar integration yet. Express the region R bound by the four curves $x = -1$, $y = -1$, $y = (x + 1)^2$, $x = y - y^3$ as a union of type I or type II regions and evaluate the integral.

$$\iint_R y \, dA .$$



- 2 Evaluate the given integral by changing to polar coordinates:

$$\iint_R 6x \, dA ,$$

where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

- 3 Use polar coordinates to find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.
- 4 a) A city near the sea is modeled by a half disk $D = \{(x, y) \mid x^2 + y^2 \leq 49, x \geq 0\}$ with center the origin and radius 7. What is the average distance of a point in D to the origin? in other words, what is the integral $\int \int_D \sqrt{x^2 + y^2} \, dx dy / \int \int_D 1 \, dx dy$.
- b) The distance to the beach is x . Find the average distance $\int \int_D x \, dx dy / \int \int_D 1 \, dx dy$ to the beach.

- 5 Evaluate the iterated integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} 9\sqrt{x^2 + y^2} \, dy \, dx .$$

Main definitions

Polar coordinates $(x, y) = (r \cos(t), r \sin(t))$ allow to describe regions bound by polar curves $(r(\theta), \theta)$.

The **average** of a quantity $f(x, y)$ over a region G is the fraction

$$\frac{\int \int_G f(x, y) dA}{\int \int_G 1 dA} .$$

To integrate in polar coordinates, we evaluate the integral

$$\int \int_R f(x, y) dx dy = \int \int_R f(r \cos(\theta), r \sin(\theta)) r dr d\theta ,$$

where R is described in polar coordinates.

Example:

To integrate $f(x, y) = x^2 + y^2$ over the region $x^2 + y^2 \leq 9, x \geq 0, y \geq 0$, we integrate

$$\int_0^{\pi/2} \int_0^3 r^2 \cdot r dr d\theta = (\pi/2)(3^4/4) = 81\pi/8 .$$