

Homework 24: Vector fields

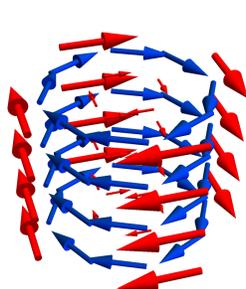
This homework is due Wednesday, 11/9 resp Thursday 11/10.

1 Match the vector fields \vec{F} with the plots labeled A-D.

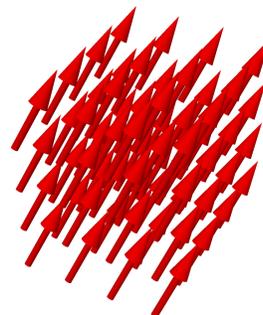
a) $\vec{F}(x, y, z) = \langle x, 2y, 3z \rangle$

b) $\vec{F}(x, y, z) = \langle y, -x, 0 \rangle$, c) $\vec{F}(x, y, z) = \langle 1, 2, 3 \rangle$

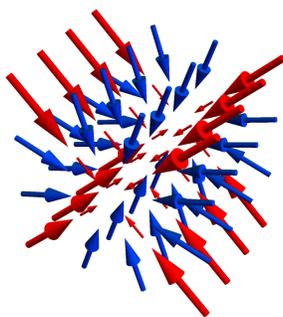
d) $\vec{F}(x, y, z) = \langle -x, 0, -z \rangle$,



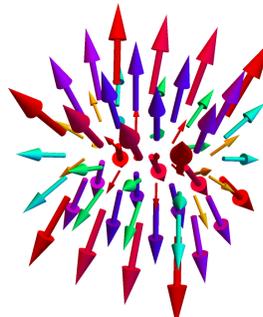
A



B



C



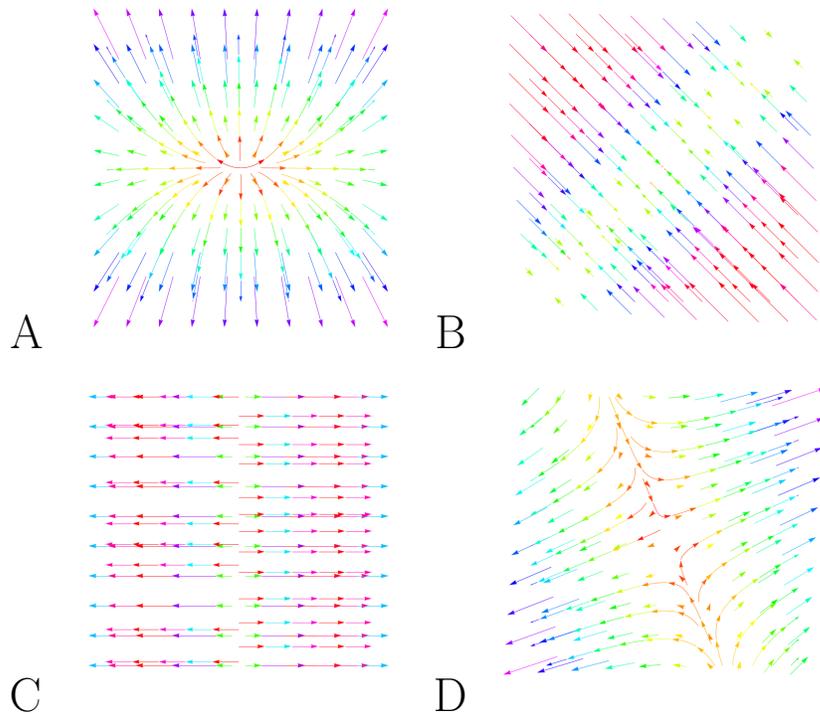
D

2 a) Compute the gradient vector field of f , where $f(x, y, z) = \log(7x - 5y + z)$ and \log is the natural log.

b) Given the vector field $\vec{F} = \langle P, Q \rangle = \langle \frac{x}{\sqrt{x^2-3y^2}} + 9x^2y + 1, 3x^3 - \frac{3y}{\sqrt{x^2-3y^2}} \rangle$. Check that $Q_x - P_y = 0$ and find a function $f(x, y)$ for which $\nabla f = \vec{F}$.

3 Match the functions f with the plots of their gradient fields labeled A – D. Give reasons for your choices.

- a) $f(x, y) = x^2 + 2y^2$, b) $f(x, y) = x(x + y)$
 c) $f(x, y) = \cos(x - y)$, d) $f(x, y) = \sin(\sqrt{1 + x^2})$



- 4 a) Sketch the vector field $\vec{F}(x, y) = \langle 2x, 4y \rangle$ and then sketch some flow lines. What shape to these flow lines appear to have?
 b) Find the flow line $\vec{r}(t)$ with $\vec{r}(0) = \langle 1, 1 \rangle$.
- 5 a) Let $\vec{F}(x, y) = \langle x^3 - y, x^5 + x \rangle$. Plot the vector field using Mathematica.
 b) Make a stream plot of the field $\vec{F}(x, y) = \langle x^3 - \sin(y) + 2y, \sin(x^5) + x^2 \rangle$. using Mathematica. If you start on the line $y = -1$, there is a watershed threshold so that if x is larger than this value the flow will go to the right and to the left, the path will go to the left. Find this value (round to the next integer).

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VectorPlot[{x+y, x^2}, {x, -2, 2}, {y, -2, 2}]
StreamPlot[{x+y, x^2}, {x, -2, 2}, {y, -2, 2}]
VectorPlot3D[{x, y, z}, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
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