Homework 31: Divergence Theorem

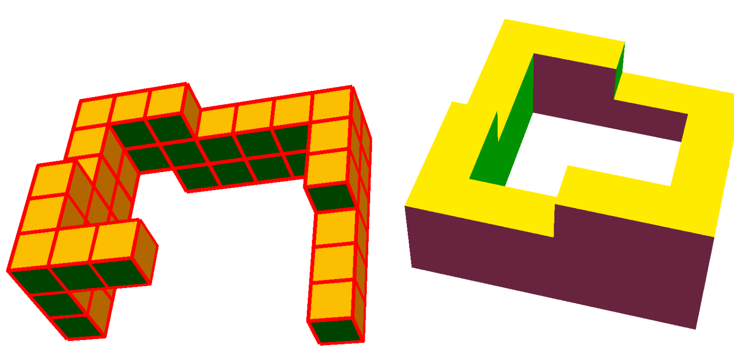
This last homework is due Friday, 12/2 rsp Thursday 12/1 in the last lecture.

1 Find the flux of the field $\vec{F}(x,y,z) = \langle 2x^2+2z^{10}, 2xy+x, 8z-y\rangle$ through the boundary of the solid bounded by paraboloid $z = 4-x^2-y^2$ and the xy-plane. by using the divergence theorem.

2 Find the flux of the vector field $\vec{F}(x,y,z) = \langle x^2y+\cos^6(y), xy^2, 2xyz+e^{\sin(x)}\rangle$ through the outwards oriented solid bound by $x=0, y=0, z=0$, and $x+2y+z=2$.

3 Evaluate the flux $\int\int_S \vec{F} \cdot d\vec{S}$ where S is the hemisphere $z = \sqrt{1-x^2-y^2}$ together with the disk $x^2+y^2 \leq 1$ in the xy plane and where $\vec{F}(x,y,z) = \langle x,y,z\rangle/\sqrt{x^2+y^2+z^2}$.

4 Find $\int\int_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x,y,z) = \langle x+\sin(y)+e^z, y+\sin(z)+e^z, z+\sin(x)+e^y\rangle$ and S is the boundary of the Escher stair solid displayed in the picture. The right picture shows the same figure from an other angle leading to the illusion. Each brick is a cube of unit length 1.



5 a) Use an integral theorem to evaluate $\int\int_S \text{curl}(\vec{F}) \cdot d\vec{S}$, where $\vec{F}(x,y,z) = \langle x^2yz, yz^2, z^3e^{xy}\rangle$, where is the part of upwards oriented surface $x^2+y^2+z^2 = 5$ that lies above the plane $z=1$.

b) Use an integral theorem to compute the line integral of $\vec{F}(x,y,z) = \langle x^3, y^5, 2z \rangle$ along the path $\vec{r}(t) = \langle \cos(t) + t^{100}\sin(17t), \sin(t) + \sin(20t), t \rangle$ from $t = 0$ to $t = 10\pi$.

Main points

Divergence Theorem.

$$\int\int\int_E \operatorname{div}(\vec{F})\, dV = \int\int_S \vec{F} \cdot d\vec{S}\ .$$

All integral theorems are incarnations of **the fundamental theorem of multivariable Calculus**

$$\int_G dF = \int_{\delta G} F$$

where dF is a **derivative** of F and δG is the **boundary** of G.