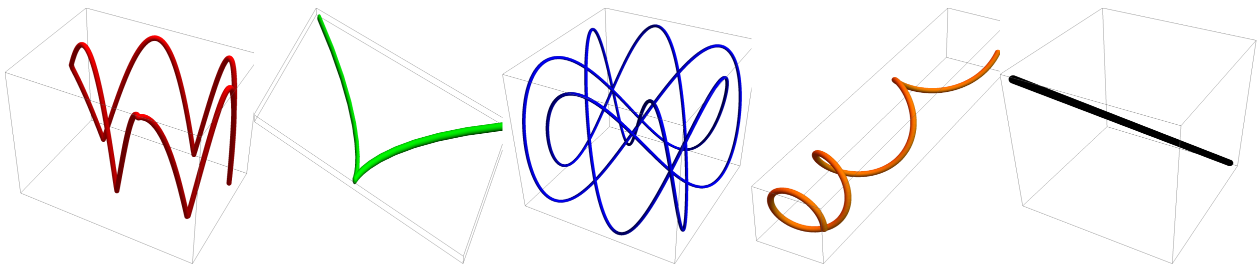
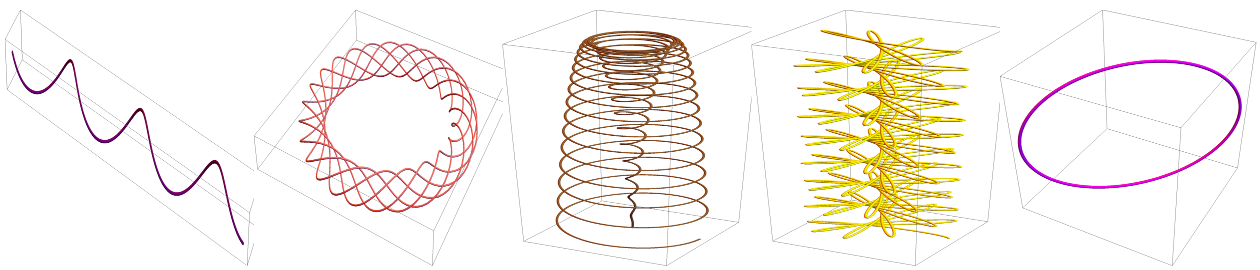
Homework 5: Parametrized curves

This homework is due Monday, 9/19 rsp Tuesday 9/20.

1 Match the curves:



I II III IV V



VI VII VIII IX X

$\vec{r}(t) =$	I-X
$\langle t, t, -t \rangle$	
$\langle t\cos(8t), t\sin(8t), t(8\pi - t) \rangle$	
$\langle (6+\cos(24t))\cos(5t), (6+\cos(24t))\sin(5t), \sin(24t) \rangle$	
$\langle \cos(40t)\cos(3t), \cos(t40)\sin(4t), t \rangle$	
$\langle \cos(t), t^2, \sin(t) \rangle$	
$\langle t, \cos(t), \sin(t) \rangle$	
$\langle t^3, t^2, 0 \rangle$	
$\langle \cos(3t), \sin(4t), \cos(7t) \rangle$	
$\langle \cos(t), \cos(t), \sin(t) \rangle$	
$\langle \lvert\cos(t)+\sin(t)\rvert, \lvert\sin(t)\rvert, \lvert\cos(5t)\rvert \rangle$	

2 Parametrize the intersection of the quartic paraboloid $z = 3x^4 - y^4$ with the elliptic cylinder $(x-1)^2/4 + y^2/9 = 1$.

3 a) Two particles travel along the space curves $\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\vec{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$. Do the particles collide? Do the particle paths intersect?
b) If $\vec{r}(t) = \langle \cos(t), 2\sin(t), 4t \rangle$, find $\vec{r}\,'(0)$ and $\vec{r}\,''(0)$. Then compute $|\vec{r}\,'(0) \times \vec{r}\,''(0)|/|\vec{r}\,'(0)|^3$. We will later call this the curvature.

4 Find the point of intersection of the two tangent lines to the curve $\vec{r}(t) = \langle \sin(\pi t), 2\sin(\pi t), \cos(\pi t) \rangle$ at the points where $t = 0$ and $t = 0.5$.

5 A particle moving along a curve $\vec{r}(t)$ has the property that $\vec{r}\,''(t) = \langle 1, 0, \sin(2t) \rangle$. We know $\vec{r}(0) = \langle 1, 1, 2 \rangle$ and $\vec{r}\,'(0) = \langle 1, 0, 0 \rangle$. What is $\vec{r}(\pi)$?

Main definitions

The parametrization of a space curve is $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$. The **image** of r is a **parametrized curve** in space. If $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a curve, then $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle = \langle \dot{x}, \dot{y}, \dot{z} \rangle$ is called the **velocity** at time t. Its length $|\vec{r}\,'(t)|$ is called **speed** and $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$ is called **unit tangent vector** or direction of motion. The vector $\vec{r}\,''(t)$ is called the **acceleration**. When knowing the acceleration and $\vec{r}\,'(0)$ and $\vec{r}(0)$ we can get back position $\vec{r}(t)$ by integration. Similarly, if we know $\vec{r}\,''(t)$ at all times and $\vec{r}(0)$ and $\vec{r}\,'(0)$, we can compute $\vec{r}(t)$ by integration.