Homework 7: Other coordinates

This homework is due Friday, 9/23 rsp Tuesday 9/27.

1 Change to cylindrical coordinates: a) $(-3\sqrt{3}, 3, 27)$ b) $(-4, 4, 10)$
Change to spherical coordinates: c) $(-5\sqrt{3}, 0, 5)$ d) $(-1, -1, \sqrt{2})$

2 a) Identity the surface $r^2 + 1 = (z-5)^2$.
b) Identify the surface given in spherical coordinates as $\rho\sin(\phi) = 2\rho\cos(\phi)$.
c) Write the surface $\sin^2(\phi) + \cos^2(\phi)/4 = 1/\rho^2$ in Cartesian coordinates.

3 a) Identify the surface $2r^2 - z^2 = 1$.
b) Identify the surface $r^2 - z = 1$.
b) Identify the surface $\cos(\phi) = \rho\sin^2(\phi)$.

4 a) Identity the surface whose equation is given in cylindrical coordinates as

$$\cos(\theta) + \sin(\theta) = 1/r \ .$$

b) Identify the surface whose equation is given in spherical coordinates as

$$\rho^2(\sin^2(\phi)\sin^2(\theta) + \cos^2(\phi)) = 16 \ .$$

5 a) Use the Mathematica command "RevolutionPlot3D" to plot the surface which is given by

$$z = (\sin(5\theta) + \cos(5\theta))e^{-r^2} \ .$$

(See example Mathematica command below). b) Use the Mathematica command "SphericalPlot3D" to plot a bumpy sphere

$$\rho(\phi,\theta) = (3 + \cos(\theta) + \sin(11\theta)\sin(13\phi)) \ .$$

with $0 \leq \phi \leq \pi$ and $0 \leq \theta < 2\pi$.

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RevolutionPlot3D[Cos[r t], {r,0,Pi},{t,0,2Pi}]
SphericalPlot3D[s+ t, {s,0,Pi},{t,0,2Pi}]
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Main definitions

A point (x, y) in the plane has the **polar coordinates** $r = \sqrt{x^2 + y^2} \geq 0, \theta = \text{arctg}(y/x)$. We have the relation $(x, y) = (r\cos(\theta), r\sin(\theta))$. Chose the arctan values in $(-\pi/2, \pi/2]$ and add π if $x < 0$ or $x = 0, y < 0$. A point (x, y, z) in space has the **cylindrical coordinates** (r, θ, z), where (r, θ) are the polar coordinates of (x, y).

A curve given in polar coordinates as $r(\theta) = f(\theta)$ is called a **polar curve**. It can in Cartesian coordinates be described as $\vec{r}(t) = \langle f(t)\cos(t), f(t)\sin(t)\rangle$.

Spherical coordinates use the distance ρ to the origin as well as two angles θ and ϕ. The first angle θ is the polar angle in polar coordinates of the xy coordinates and ϕ is the angle between the vector $\vec{OP}$ and the z-axis. The relation is $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$. We have

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\phi), \\ y &= \rho\sin(\theta)\sin(\phi), \\ z &= \rho\cos(\phi) \end{aligned}$$