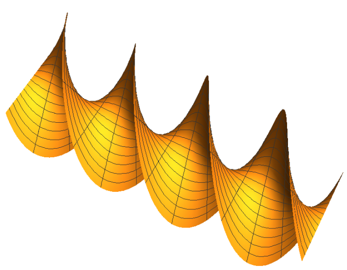
Homework 8: Parametrized surfaces

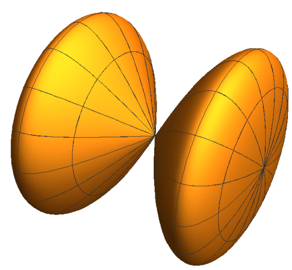
This homework is due Monday, 9/26 rsp Tuesday 9/27.

1 a) Identify the surface $\vec{r}(u,v) = \langle \sqrt{v}\sin(u), v, \sqrt{v}\cos(u) \rangle$.
b) Identity the surface $\vec{r}(s,t) = \langle s^6 - t^6, t^3, s^3 \rangle$.

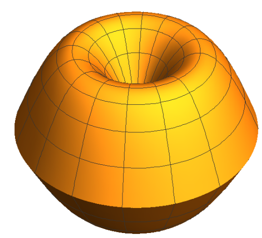
2 Match the following surfaces:



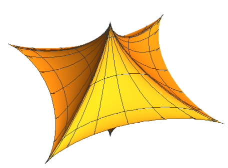
I



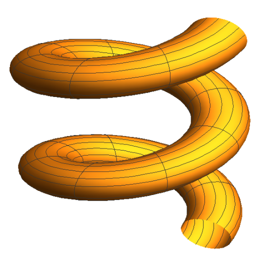
II



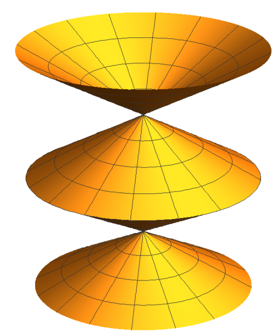
III



IV



V



VI

$\vec{r}(u,v) =$	I-VI
$\langle (3+\cos(v))\cos(u), (3+\cos(v))\sin(u), \sin(v)+u \rangle$	
$\langle \sin(v), \cos(u)\sin(2v), \sin(u)\sin(2v) \rangle$	
$\langle \|1-\|u\|\|\cos(v), \|1-\|u\|\|\sin(v), u \rangle$	
$\langle u\cos(v), u\sin(v), 2\sin(u) \rangle$	
$\langle \cos(u)^3\cos(v)^3, \sin(u)^3\cos(v)^3, \sin(v)^3/2 \rangle$	
$\langle v, u\cos(v), u\sin(v) \rangle$	

3 Find parametric equations for the surface obtained by rotating the curve $x = f(y) = 4y^2 - y^5, -2 \le y \le 2$, about the y-axis and use the graph of f to make a picture of the surface.

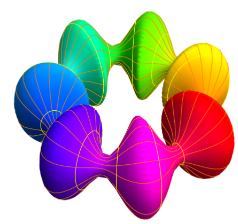
4 The surface with parametric equations

$$\vec{r}(\theta, r) = \langle 2\cos(\theta) + r\cos(\theta/2), 2\sin(\theta) + r\cos(\theta/2), r\sin(\theta/2)\rangle$$

with $-1/2 \le r \le 1/2$ and $0 \le \theta \le 2\pi$ is a **Möbius strip**. Sketch this surface.

5 Find a parametrisation of the **bumpy torus**, given as the set of points which have distance $5 + 2\cos(7\theta)$ from the circle $\langle 10\cos(\theta), 10\sin(\theta), 0\rangle$, where θ is the angle occurring in cylindrical and spherical coordinates.

Hint: Use r, the distance of a point (x, y, z) to the z-axis. This distance is $r = (10 + (5 + 2\cos(7\theta))\cos(\phi))$ if ϕ is the angle you see on Figure 1. You can read off from the same picture also $z = (5 + 2\cos(7\theta))\sin(\phi)$. To finish the parametrization problem, translate back to Cartesian coordinates.



Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle \ ,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

Plane: $\vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$
Sphere $\vec{r}(u, v) = \langle \rho\cos(u)\sin(v), \rho\sin(u)\sin(v), \rho\cos(v)\rangle$.
Graph: $\vec{r}(u, v) = \langle u, v, f(u, v)\rangle$
Surface of revolution: $\vec{r}(u, v) = \langle g(v)\cos(u), g(v)\sin(u), v\rangle$