Name:

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TTH 11:30 Sebastian Vasey

- Start by printing your name in the above box and **check your section** in the box to the left.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader cannot be given credit.

- **Show your work**. Except for problems 1-3,8, we need to see **details** of your computation.

- All functions can be differentiated arbitrarily often unless otherwise specified.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

1) ☐ T ☐ F   If $\vec{r}(t)$ is a space curve satisfying $\vec{r}\,'(0) = 0$ and $f(x, y, z)$ is a function of three variables then $\frac{d}{dt}f(\vec{r}(t)) = 0$ at $t = 0$.

2) ☐ T ☐ F   The integral $\int \int_R 1\, dxdy$ is the area of the region R in the xy-plane.

3) ☐ T ☐ F   If $f(x, y)$ is a linear function in x, y, then $D_{\vec{u}}f(x, y)$ is independent of $\vec{u}$.

4) ☐ T ☐ F   If $f(x, y)$ is a linear function in x, y, then $D_{\vec{u}}f(x, y)$ is independent of (x, y).

5) ☐ T ☐ F   If $(0, 0)$ is a saddle point of $f(x, y)$ it is possible that $(0, 0)$ is a minimum of $f(x, y)$ under the constraint $x = y$.

6) ☐ T ☐ F   The equation $f_{xy}(x, y) = 0$ is an example of a partial differential equation.

7) ☐ T ☐ F   The linearization of $f(x, y) = 4 + x^3 + y^3$ at $(x_0, y_0) = (0, 0)$ is $L(x, y) = 4 + 3x^2 + 3y^2$.

8) ☐ T ☐ F   Assume $(1, 1)$ is a saddle point of $f(x, y)$. Then $D_{\vec{v}}f(1, 1)$ takes both positive and negative values as $\vec{v}$ varies over all directions.

9) ☐ T ☐ F   The integral $\int_{\pi/2}^{\pi} \int_0^2 r\, drd\theta$ is equal to π.

10) ☐ T ☐ F   If $|\nabla f(0, 0)| = 1$, then there is a direction in which the slope of the graph of f at $(0, 0)$ is 1.

11) ☐ T ☐ F   The vector $\nabla f(a, b)$ is a vector in space orthogonal to surface defined by $z = f(x, y)$ at the point (a, b).

12) ☐ T ☐ F   If $f(x, y, z) = 1$ defines y as a function of x and z, then $\partial y(x, z)/\partial x = -f_x(x, y, z)/f_y(x, y, z)$.

13) ☐ T ☐ F   In a constrained optimization problem it is possible that the Lagrange multiplier λ is 0.

14) ☐ T ☐ F   The area $\int \int_R |\vec{r}_u \times \vec{r}_v|\, dudv$ of a surface is independent of the parametrization.

15) ☐ T ☐ F   The function $f(x, y) = x^6 + y^6 - x^5$ has a global minimum in the plane.

16) ☐ T ☐ F   The area of a graph $z = f(x, y)$ where (x, y) is in a region R is the integral $\int \int_R |f_x \times f_y|\, dxdy$.

17) ☐ T ☐ F   The gradient of a function $f(x, y)$ of two variables can be written as $\langle D_{\vec{i}}f(x, y), D_{\vec{j}}f(x, y)\rangle$, where $\vec{i} = \langle 1, 0\rangle$ and $\vec{j} = \langle 0, 1\rangle$.

18) ☐ T ☐ F   The length of the gradient of f at a critical point is positive if the discriminant $D(x, y) = f_{xx}f_{yy} - f_{xy}^2$ is strictly positive.

19) ☐ T ☐ F   If $f(0, 0) = 0$ and $f(1, 0) = 2$ then there is a point on the line segment between $(0, 0)$ and $(1, 0)$, where the gradient has length at least 2.

20) ☐ T ☐ F   The tangent plane of the surface $-x^2 - y^2 + z^2 = 1$ at $(0, 0, 1)$ intersects the surface at exactly one point.

Problem 2) (10 points)

a) (6 points) Match the regions with the integrals. Each integral matches exactly one region $A - F$.

A

B

C

D

E

F
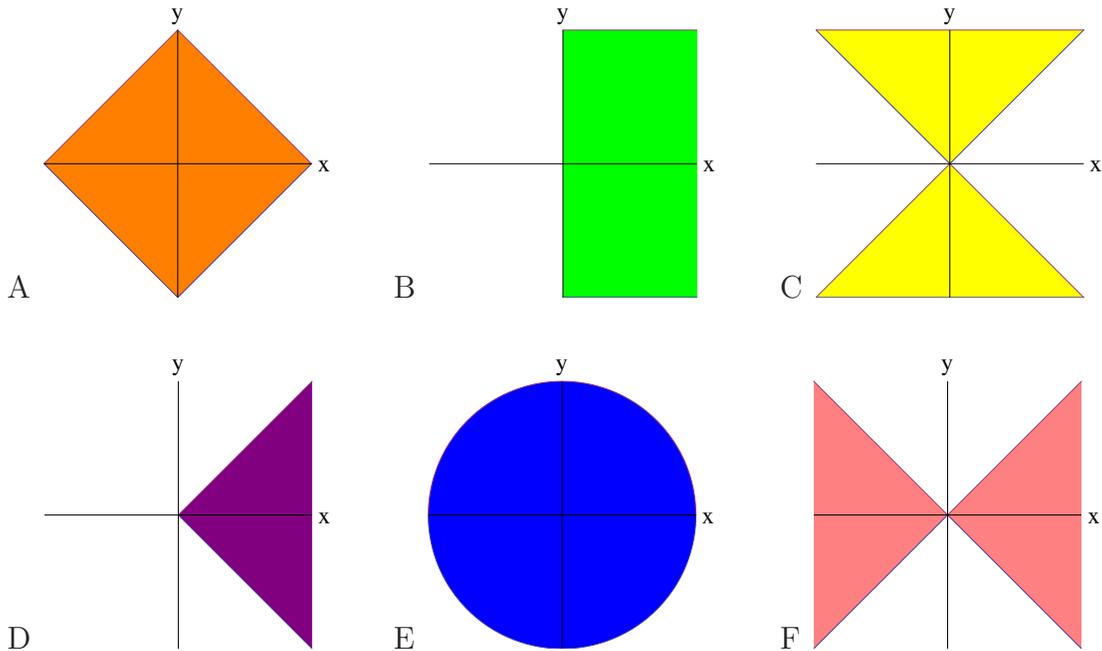
Enter A-F	Integral
	$\int_{-\pi}^{\pi} \int_{-|y|}^{|y|} f(x,y)\ dxdy$
	$\int_{-\pi}^{\pi} \int_{0}^{\pi} f(r,\theta) r\ drd\theta$
	$\int_{-\pi}^{\pi} \int_{-|x|}^{|x|} f(x,y)\ dydx$
	$\int_{0}^{\pi} \int_{-|x|}^{|x|} f(x,y)\ dydx$
	$\int_{-\pi}^{\pi} \int_{0}^{\pi} f(x,y)\ dxdy$
	$\int_{-\pi}^{\pi} \int_{-\pi+|x|}^{\pi-|x|} f(x,y)\ dydx$

b) (4 points) Name the partial differential equations correctly. Each equation appears once to the left.

Fill in 1-4	Order
	Burgers
	Transport
	Heat
	Wave

Equation Number	PDE
1	$u_x - u_y = 0$
2	$u_{xx} - u_{yy} = 0$
3	$u_x - u_{yy} = 0$
4	$u_x + u u_x - u_{xx} = 0$

Problem 3) (10 points)

(10 points) Let's label some points in the following contour map of a function $f(x, y)$ indicating the height of a region. The arrows indicate the gradient $\nabla f(x, y)$ at the point. Each of the 11 selected points appears each exactly once.

| Enter A-K	description
	a local minimum of $f(x, y)$ inside the circle
	a saddle point of $f(x, y)$ inside the circle
	a point, where $f_x \neq 0$ and $f_y = 0$
	a point, where $f_x = 0$ and $f_y > 0$
	a point, where $f_x = 0$ and $f_y < 0$
	a point on the circle, where $D_{\vec{v}} f = 0$ with $\vec{v} = \langle 2, -1 \rangle / \sqrt{5}$.
	the lowest point on the circle
	the highest point on the circle
	the local but not global maximum inside or on the circle
	the global maximum inside or on the circle
	the steepest point inside the circle
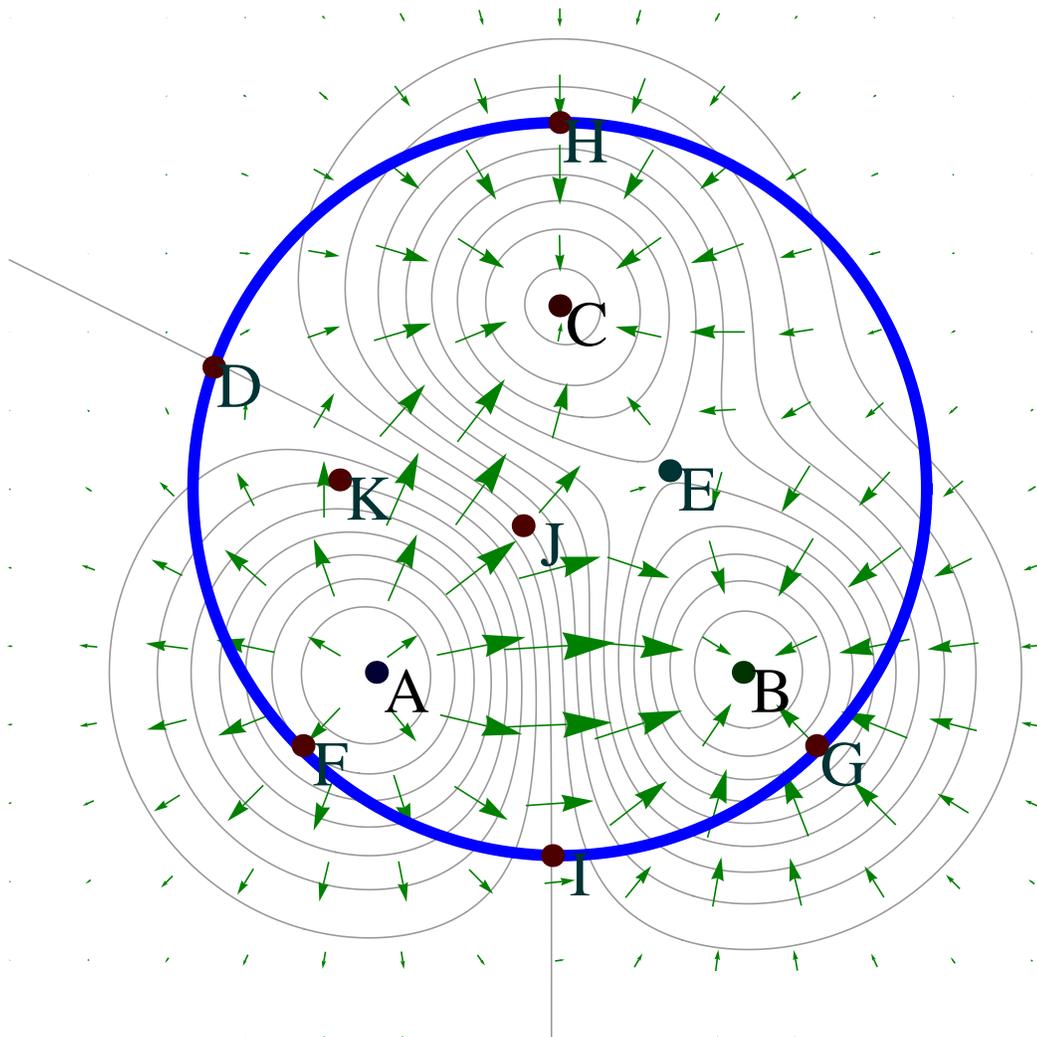
Problem 4) (10 points)

On October 30, 2012, the wind speed of Hurricane Sandy was given by the function

$$f(x, y) = 60 - x^3 + 3xy + y^3 .$$

Classify the critical points (maxima, minima and saddle points) of this function. Compute also the values of f at these points.
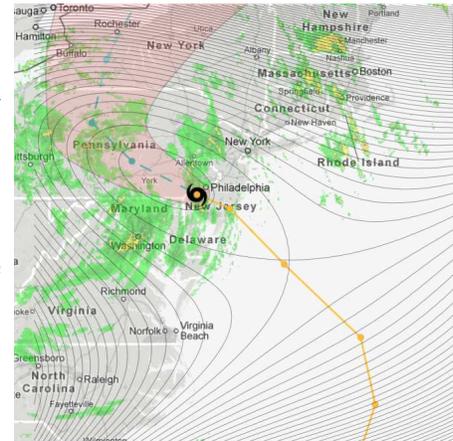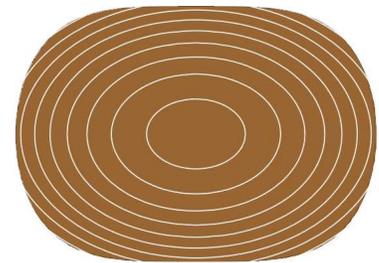
Problem 5) (10 points)

Use the second derivative test and the method of Lagrange multipliers to find the global maximum and minimum of the sugar concentration $f(x, y) = 10 + x^2 + 2y^2$ on a cake given by

$$g(x, y) = x^4 + 4y^2 \leq 4 .$$

Note that this means you have to look both inside the cake and on the boundary.

Problem 6) (10 points)

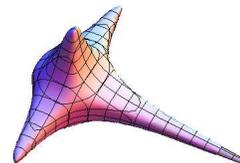a) (5 points) A seed of "Tribulus terrestris" has the shape

$$x^2 + y^2 + z^2 + x^4y^4 + x^4z^4 + y^4z^4 - 9z = 21$$

Find the tangent plane at $(1, 1, 2)$.

b) (5 points) The seed intersects with the xy-plane in a curve

$$x^2 + y^2 + x^4y^4 = 21 \ .$$

Find the tangent line to this curve at $(1, 2)$.

Problem 7) (10 points)

Let $f(x, y)$ model the time that it takes a rat to complete a maze of length x given that the rat has already run the maze y times. We know $f_y(10, 20) = -5$ and $f_x(10, 20) = 1$ as well as $f(10, 20) = 45$. Use this to estimate $f(11, 18)$.

Problem 8) (10 points)

a) (5 points) Find the double integral

$$\int\int_R x \, dydx \ ,$$

where R is the region obtained by intersecting $x \leq |y|$ with $x^2 + y^2 \leq 1$.

b) (5 points) The square $\sin^2(x)/x^2$ of the sync function $\sin(x)/x$ does not have a known antiderivative. Compute nevertheless the integral
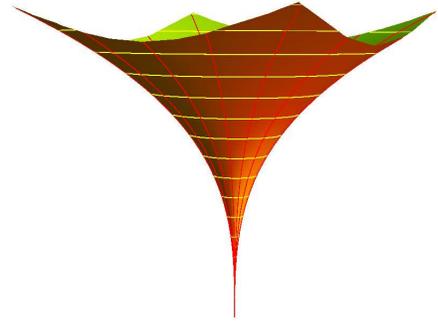
$$\int_0^{\pi^2/4} \int_{\sqrt{y}}^{\pi/2} \frac{\sin^2(x)}{x^2} \, dx \, dy \ .$$

Problem 9) (10 points)

Find the surface area of the surface of revolution $x^2 + y^2 = z^6$ where $0 \le z \le 1$. The surface is parametrized by

$$\vec{r}(t, z) = \langle z^3 \cos(t), z^3 \sin(t), z \rangle$$

with $0 \le t \le 2\pi$ and $0 \le z \le 1$.

Problem 10) (10 points)

It turns out that there is only one way to identify zombies: throw two difficult integrals at them and see whether they can solve them. Prove that you are not a zombie!

a) (6 points) Find the integral

$$\int_0^1 \int_{\sqrt{y}}^{y^2} \frac{x^7}{\sqrt{x - x^2}} \, dx \, dy .$$

b) (4 points) Integrate

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)^{10} \, dx dy .$$