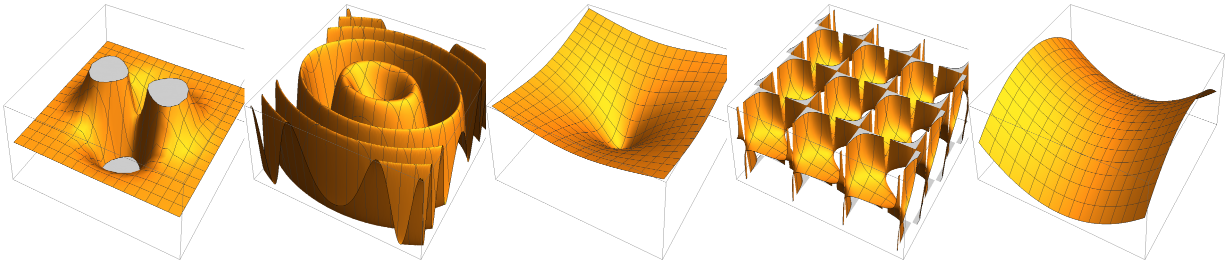
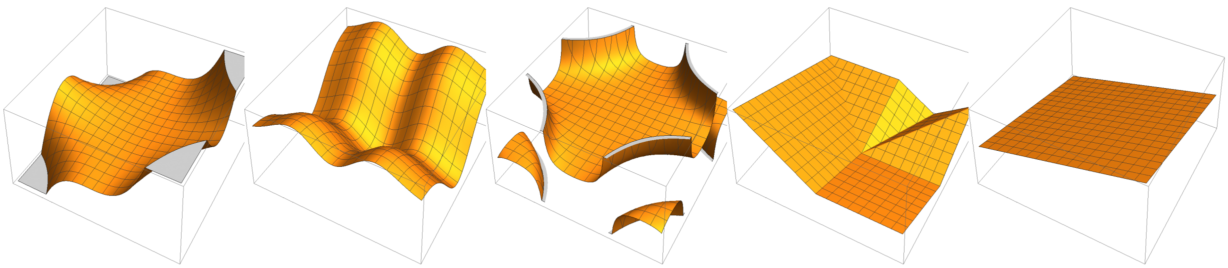
Homework 4: Functions of 2 and 3 variables

This homework is due Friday, 9/15 rsp Tuesday 9/19/2017.

1 Match the following graphs with the functions $f(x, y)$.



I II III IV V



VI VII VIII IX X

$f(x,y) =$	I-X
$\log(x^2 + y^2 + 1)$	
$x^2 - y^2$	
$\|x - \|y - \|x\|\|\|$	
$x^2 + x^3y^2$	
$\exp(-x^2 - y^2)(x^2 - y^2)$	

$f(x,y) =$	I-X
$\tan(x)/\tan(y)$	
$\sin(x^2 + 2y^2)$	
$\exp(-x^2)x^2 - \exp(-y^2)$	
$x - y$	
$\sec(xy)$	

Solution:

$f(x,y) =$	I-X
$\log(x^2 + y^2 + 1)$	III
$x^2 + y^2$	V
$\|x - \|y - \|x\|\|\|$	IX
$x^2 + x^3y^2$	VI
$\exp(-x^2 - y^2)(x^2 - y^2)$	I

$f(x,y) =$	I-X
$\tan(x)/\tan(y)$	IV
$\sin(x^2 + 2y^2)$	II
$\exp(-x^2)x^2 - \exp(-y^2)$	VII
$-y$	X
$\sec(xy)$	VIII

2 Define the function $f(x,y) = 1 + x^2y^2e^{-x^2-y^2}$.

a) What are the traces, the intersections with the coordinate planes? Draw also generalized traces, the intersection of the graph with plane $y = 1$, $y = -1$, $x = 1$, $x = -1$.

b) Use the information obtained in a) to plot the graph of the function. It is a function c) Find the domain and range of the function

$f(x,y) = \sqrt{(}x^2 - y^2)$ and plot the graph, where defined.

Solution:

a) If you look at traces, you see for $y = 1$ that the function has two bumps. The traces are lines or empty.

b) There are 4 mountains with mountain tops at the vertices of a square.

c) The function is defined only if $x^2+y^2 < 1$ and $-1 < x^2-y^2 < 1$. This is a region bound by two hyperbola, a circle.

3 a) Plot both the graph and contour map of the function $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$

b) Plot both the graph and contour map of the function $f(x,y) = \frac{xy}{x^4+y^4}$. Its ok to use technology.

Solution:

a) The contours are lines through the origin. b) Also this needs to be plotted with help. We see klover type

4 Find an equation for the surface consisting of all points P for which the distance from P to the x-axes is twice the distance from P to the yz-plane. Identify the surface.

Solution:
The distance from a point to the x-axis is $\sqrt{y^2+z^2}$. The distance to the yz-plane is $|x|$. The surface consisting of all these points is defined by $\sqrt{y^2+z^2} = 2|x|$. This is a perfectly acceptable answer, but simplifying is nicer: square both sides and we get $y^2 + z^2 = 4x^2$. From this equation, we can determine that this is a cone whose axis is the x-axis.

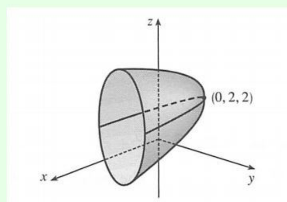
5 a) Draw the surface $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.
b) Draw the surface $x - z^2 + y^2 + 4y = 1$.
Try to make a clear, what surface is displayed. Your drawing does not have to be to scale.

Solution:

(a) Rearranging this equation and completing the square yields:

$$\begin{aligned} 4y^2 - 16y + z^2 - 4z &= x - 20 \\ 4(y^2 - 4y + 4) + z^2 - 4z + 4 &= x - 20 + 16 + 4 \\ 4(y-2)^2 + (z-2)^2 &= x \end{aligned}$$

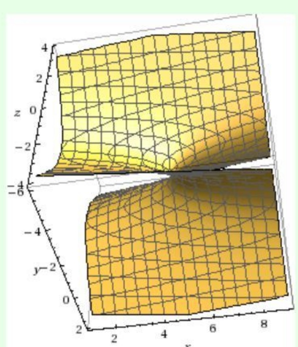
We can see that this is the equation of an elliptic paraboloid with vertex (0, 2, 2) that opens in the positive-x direction along the axis $y = 2, z = 2$. The paraboloid is stretched in the y direction by a factor of $\frac{1}{2}$.



(b) Rearranging this equation and completing the square yields:

$$\begin{aligned} x - z^2 + y^2 + 4y + 4 &= 5 \\ x - z^2 + (y+2)^2 &= 5 \\ x &= z^2 - (y+2)^2 + 5 \end{aligned}$$

We can see that this equation describes a hyperbolic paraboloid (i.e. a saddle). The saddle is centered at (0, -2, 0).



Main definitions

The **domain** D of a function $f(x,y)$ is the set of points where f is defined, the **range** is $\{f(x,y) \mid (x,y) \in D\}$. The **graph** of $f(x,y)$ is the surface $\{(x,y,f(x,y)) \mid (x,y) \in D\}$ in $\mathbb{R}^3$. The set $f(x,y) = c = \text{const}$ is **contour curve** or **level curve** of f. The collection of contour curves $\{f(x,y) = c\}$ is the **contour map** of f. A function of three variables $g(x,y,z)$ can be visualized by **contour surfaces** $g(x,y,z) = c$, where c is constant. **Traces**, the intersections of the surfaces with the coordinate planes help to draw them. Examples: The elliptic paraboloid $z-x^2-y^2 = 0$ and hyperboloid $z-x^2+y^2 = 0$ are examples of graphs $z - f(x,y) = 0$. The one sheeted hyperboloid $x^2+y^2-z^2 = 1$ and two sheeted hyperboloid $x^2+y^2-z^2 = -1$ or cylinder $x^2+y^2 = 1$ are examples of surfaces of revolution $x^2+y^2-g(z) = 0$.