Homework 7: Other coordinates

This homework is due Friday, 9/22 rsp Tuesday 9/26.

1 Change to cylindrical coordinates: a) $(x, y, z) = (-3\sqrt{3}, 3, 27)$ b) $(x, y, z) = (-4, 4, 10)$
Change to spherical coordinates: c) $(x, y, z) = (-5\sqrt{3}, 0, 5)$ d) $(x, y, z) = (-1, -1, \sqrt{2})$

Solution:
We compute:

(a) $r = \sqrt{x^2 + y^2} = 6$, $\theta = \pi/3 + \pi/2$, $z = z = 27$.

(b) $r = \sqrt{32}$, $\theta = 3\pi/4$ and $z = 10$.

(c) $\rho = \sqrt{x^2 + y^2 + z^2} = 10$, $\theta = \pi$, $\phi = \arccos(z/\rho) = \pi/3$.

(d) $\rho = 2$, $\theta = 5\pi/4$ and $\phi = \pi/4$.

2 a) Identity the surface $r^2 = 1 + (z - 5)^2$.
b) Identify the surface given in spherical coordinates as $\rho \sin(\phi) = 2\rho \cos(\phi)$.
c) Write the surface $\sin^2(\phi) + \cos^2(\phi)/4 = 1/\rho^2$ in Cartesian coordinates.

Solution:

(a) It is a (one-sided) hyperboloid $x^2 + y^2 - (z - 5)^2 = 1$.

(b) This translates to $r = 2z$ in cylindrical coordinates which is a cone.

(c) Multiply with ρ^2. This is an ellipsoid $x^2 + y^2 + z^2/4 = 1$.

3 a) Identify the surface $2r^2 - z^2 = 1$.
b) Identify the surface $r^2 - z = 1$.
b) Identify the surface $\cos(\phi) = \rho \sin^2(\phi)$.

Solution:
(a) Converting to rectangular coordinates, we find $2(x^2 + y^2) - z^2 = 1$ which is a one-sheeted hyperboloid since 1 is positive.
(b) This is an elliptic paraboloid.
(c) The surface is again an elliptic paraboloid. To see this multiply both sides by ρ to obtain the equation $z = x^2 + y^2$.

4 a) Identity the surface whose equation is given in cylindrical coordinates as

$$\cos(\theta) + \sin(\theta) = 1/r \ .$$

b) Identify the surface whose equation is given in spherical coordinates as

$$\rho^2(\sin^2(\phi)\sin^2(\theta) + \cos^2(\phi)) = 16 \ .$$

Solution:
(a) It is a plane: if we multiply both sides by r, we get the equation $r\cos\theta + r\sin\theta = 1$, i.e $x + y = 1$.
(b) In rectangular coordinates, the equation is $y^2 + z^2 = 16$ which is a cylinder.

5 a) Use the Mathematica command "RevolutionPlot3D" to plot the surface which is given by

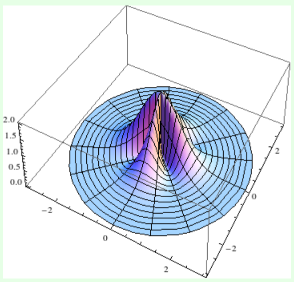
$$z = (\sin(7\theta) + \cos(7\theta))e^{-r^2} \ .$$

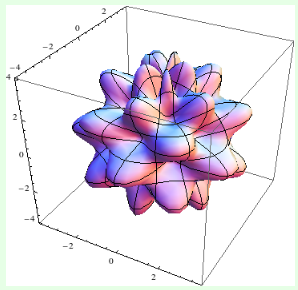
(See example Mathematica command below). b) Use the Mathematica command "SphericalPlot3D" to plot a bumpy sphere

$$\rho(\phi, \theta) = (2 + \cos(3\phi) + \sin(5\theta)) \ .$$

with $0 \leq \phi \leq \pi$ and $0 \leq \theta < 2\pi$.

Solution:





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RevolutionPlot3D[Cos[r+t], {r,0,Pi},{t,0,2Pi}]
SphericalPlot3D[s+ t, {s,0,Pi},{t,0,2Pi}]
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Main definitions

A point (x, y) in the plane has **polar coordinates** $r = \sqrt{x^2 + y^2} \geq 0, \theta = \arctan(y/x)$. We have $(x, y) = (r\cos(\theta), r\sin(\theta))$. We chose arctan values in $(-\pi/2, \pi/2]$ and add π if $x < 0$ or $x = 0, y < 0$. This gives a value in $\theta \in [0, 2\pi)$. A point (x, y, z) in space has the **cylindrical coordinates** (r, θ, z), where (r, θ) are the polar coordinates of (x, y). A curve given in polar coordinates as $r(\theta) = f(\theta)$ is called a **polar curve**. It can in Cartesian coordinates be described as $\vec{r}(t) = \langle f(t)\cos(t), f(t)\sin(t)\rangle$. **Spherical coordinates** use the distance ρ to the origin as well as two angles θ and ϕ. The first angle θ is the polar angle in polar coordinates of the xy coordinates and ϕ is the angle between the vector $\vec{OP}$ and the z-axis. The relation is $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$. We have

$$\begin{aligned} x &= \rho\cos(\theta)\sin(\phi), \\ y &= \rho\sin(\theta)\sin(\phi), \\ z &= \rho\cos(\phi) \end{aligned}$$