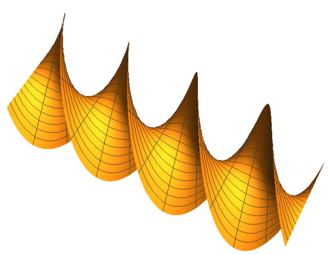
Homework 8: Parametrized surfaces

This homework is due Monday, 9/25 rsp Tuesday 9/26.

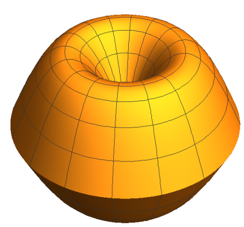
1 a) Identify the surface $\vec{r}(u,v) = \langle \sqrt{v}\sin(u), v\cos^2(u), \sqrt{v}\rangle$.
b) Identity the surface $\vec{r}(s,t) = \langle t^4, s^{12} + t^8, s^6\rangle$.

Solution:
a) It is a hyperbolic paraboloid $x^2 + y = z^2$.
b) It is an elliptic paraboloid because $y = x^2 + z^2$.

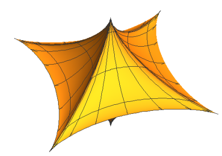
2 Match the following surfaces:



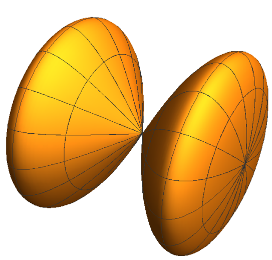
I



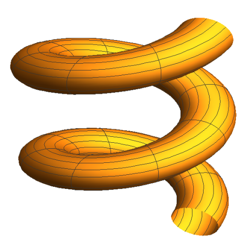
II



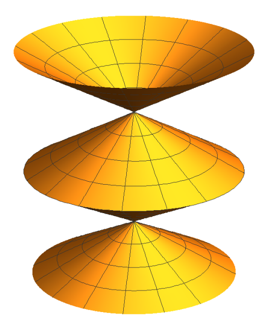
III



IV



V



VI

$\vec{r}(u,v) =$	I-VI
$\langle (3+\cos(v))\cos(u), (3+\cos(v))\sin(u), \sin(v)+u \rangle$	
$\langle \sin(v), \cos(u)\sin(2v), \sin(u)\sin(2v) \rangle$	
$\langle \lvert 1-\lvert u\rvert\rvert \cos(v), \lvert 1-\lvert u\rvert\rvert \sin(v), u \rangle$	
$\langle u\cos(v), u\sin(v), 2\sin(u) \rangle$	
$\langle \cos(u)^3\cos(v)^3, \sin(u)^3\cos(v)^3, \sin(v)^3/2 \rangle$	
$\langle v, u\cos(v), u\sin(v) \rangle$	

Solution:
V,IV, VI, II, III,I

3 Find parametric equations for the surface obtained by rotating the curve $x = f(y) = 4y^2 - y^5, -2 \leq y \leq 2$, about the y-axis and use the graph of f to make a picture of the surface.

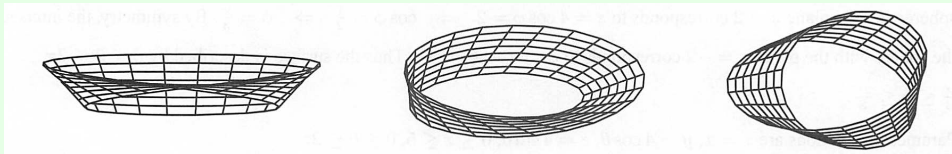
Solution:
Letting θ be the angle of rotation about the y-axis, we can see that the xz plane cross-sections are circles. Therefore, we can write the parametrization $x = (4y^2 - y^5)\cos\theta$, $y = y$, $z = (4y^2 - y^5)\sin\theta$, $-2 \leq y \leq 2$, $0 \leq \theta \leq 2\pi$.

4 Given an integer or half integer k, we can draw the surface with parametric equations

$$\vec{r}(u,v) = \langle (2+v\cos(ku))\cos(u), (2+v\cos(ku))\sin(u), -v\sin(ku) \rangle$$

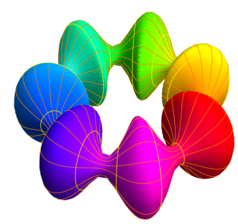
with $-1 \leq v \leq 1$ and $0 \leq u \leq 2\pi$ is called the k-ribbon. Sketch the surface in two cases, $k = 1/2$ (the Moebius strip) and $k = 3$.

Solution:



5 Find a parametrisation of the **bumpy torus**, given as the set of points which have distance $5 + 2\cos(7\theta)$ from the circle $\langle 10\cos(\theta), 10\sin(\theta), 0\rangle$, where θ is the angle occurring in cylindrical and spherical coordinates.

Hint: Use r, the distance of a point (x, y, z) to the z-axis. This distance is $r = (10 + (5 + 2\cos(7\theta))\cos(\phi))$ if ϕ is a suitable angle. Make a picture to see also $z = (5 + 2\cos(7\theta))\sin(\phi)$. To finish the parametrization problem, translate back to Cartesian coordinates.



Solution:
$\vec{r}(\theta, \phi) = \langle (10 + (5 + 2\cos(7\theta))\cos(\phi))\cos(\theta), (10 + (5 + 2\cos(7\theta))\cos(\phi))\sin(\theta), (5 + 2\cos(7\theta))\sin(\phi)\rangle$.

Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle \ ,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

Plane: $\vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$

Sphere $\vec{r}(u,v) = \langle \rho\cos(u)\sin(v), \rho\sin(u)\sin(v), \rho\cos(v)\rangle$.
Graph: $\vec{r}(u,v) = \langle u, v, f(u,v)\rangle$
Surface of revolution: $\vec{r}(u,v) = \langle g(v)\cos(u), g(v)\sin(u), v\rangle$