

1 Math21a, Fall 1998

Additional Questions – Sample Midterm 2

Question 1. [Midterm 2, Fall 1992] Consider the vector field

$$\mathbf{F}(x, y) = (y^4 - 2y + 5)\mathbf{i} + (4y^3x + \cos y)\mathbf{j}$$

Let C be the upper half of the circle of radius 3 about the origin, oriented counterclockwise. Compute $\int_C \mathbf{F} \cdot \mathbf{T} ds$.

Question 2. [Final Exam, Fall 1992] The vector field

$$\mathbf{F}(x, y) = 2x\mathbf{i} + (2y + 2)\mathbf{j}$$

is a conservative vector field.

- Find the potential function $f(x, y)$ for which $f(0, 0) = 0$.
- Let P be the point $(2, 1)$. Find the closed curve C such that for every point Q on C , $\int_P^Q \mathbf{F} \cdot \mathbf{T} ds = 0$, and for every point R not on C , $\int_P^R \mathbf{F} \cdot \mathbf{T} ds \neq 0$.
- Sketch the curve C from part (b) on the xy -plane.
- The curve C found in part (b) is the boundary of some area Ω in the xy -plane. Let W be some point in Ω , but not on C . Is $\int_P^W \mathbf{F} \cdot \mathbf{T} ds$ positive, negative or zero? Explain your reasoning clearly, carefully and completely.
- Does $f(x, y)$ take on an absolute minimum or an absolute maximum anywhere in Ω ? (Note: Ω includes the curve C .) If so, where? If not, explain how you know.

Question 3. [Final Exam, Spring 1993] Let C be the upper half circle $y = \sqrt{4 - x^2}$ oriented counterclockwise and let $\mathbf{F}(x, y) = (e^x - y)\mathbf{i} + (e^{y^3} + x)\mathbf{j}$. Find $\int_C \mathbf{F} \cdot \mathbf{T} ds$.

Question 4. [Midterm 2, Spring 1991] Let \mathbf{F} be the vector field $4y\mathbf{i} + (e^{3y} + 6x)\mathbf{j}$. You are told that $\int_C \mathbf{F} \cdot \mathbf{T} ds = 6$ (for some closed curve C). Find the area of the region enclosed by C .

Question 5. [Final Exam, Spring 1987] Let $\mathbf{F} = (x^2 + 2xy - 5y)\mathbf{i} + (x^2 + y^2 + kx + 5)\mathbf{j}$.

- For what values of k is \mathbf{F} conservative?
- Let C be a simple closed curve in the plane, traversed counterclockwise. If C has length 4 and encloses a region of area 12, find $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Find this line integral for a general k ; not just for the value you found in a.)