1.5. LINE INTEGRALS, FLOW INTEGRALS AND FLUX INTEGRALS (14.1, 14.2)

What?	Calculate?	Orientation?	Meaning? (Example)
Line integral $\int_C f\,ds$ (2D or 3D)	$\int_a^b f(r(t)) \left\|\frac{dr}{dt}\right\| dt$	C is not oriented. Make sure $a \le b$, reversing C if necessary.	If f is mass density (Kg/m), you get TOTAL MASS.
Flow along C $\int_C (F \cdot T)\,ds$ (2D or 3D)	$\int_a^b \left(F \cdot \frac{dr}{dt}\right) dt$ $\int_a^b \left(M\frac{dx}{dt} + N\frac{dy}{dt} + P\frac{dz}{dt}\right) dt$	C is oriented. You can have $a > b$, but make sure C goes from $t = a$ to $t = b$.	If F is a force field and C is a trajectory, you get WORK.
Flux across C $\int_C (F \cdot N)\,ds$ (2D only)	$\int_a^b \left(F \cdot \left(\frac{dr}{dt} \times k\right)\right) dt$ $\int_a^b \left(M\frac{dy}{dt} - N\frac{dx}{dt}\right) dt$	C is oriented. N points to your right as you walk along C. If C is closed, the convention is to orient C counterclockwise, so N points outwards.	If F is a velocity field (m/s) and C is a "membrane", you get AREA OF FLUID passing through C per unit of time (in m^2/s).

- More notations: the flow integral of $\mathbf{F}$ along an oriented curve C

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

measures how much the vector field "agrees" with C. If $\mathbf{F} = M(x,y,z)\mathbf{i} + N(x,y,z)\mathbf{j} + P(x,y,z)\mathbf{k}$, the following notation might be used

$$\int_C M(x,y,z)dx + N(x,y,z)dy + P(x,y,z)dz$$

- CIRCULATION is an alternative name for the flow integral when C is CLOSED.