## 1.6. GRADIENT FIELDS, POTENTIAL FUNCTIONS, CONSERVATIVE FIELDS (14.3)

- FUNDAMENTAL THEOREM OF CALCULUS FOR LINE INTEGRALS

$$\int_C grad\, f \cdot d\mathbf{r} = f(B) - f(A)$$

where $C$ is ANY curve joining $A$ to $B$. Since the line integral above only depends on the endpoints of $C$, we call gradient vector fields PATH-INDEPENDENT or CONSERVATIVE.

- EXACTNESS TEST: Suppose that $\mathbf{F} = M(x,y,z)\mathbf{i} + N(x,y,z)\mathbf{j} + P(x,y,z)\mathbf{k}$ is defined on an open region $D$ with no holes, and that $M$, $N$ and $P$ have continuous first derivatives. Then:

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} = 0 \Longleftrightarrow \mathbf{F} \text{ is conservative}$$

## 1.7. GREEN'S THEOREM (14.4)

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$$\int_C \mathbf{F}.d\mathbf{r} = \int_C M\,dx + N\,dy = \int\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA \,(\text{curl form})$$

$$\int_C \mathbf{F}.(d\mathbf{r} \times \mathbf{k}) = \int_C M\,dy - N\,dx = \int\int_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA \,(\text{divergence form})$$

for any CLOSED curve $C$, oriented COUNTER-CLOCKWISE enclosing a region $R$.

- CURL or CIRCULATION DENSITY of a 2D vector field $\mathbf{F} = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ is defined as the following scalar function

$$curl\,\mathbf{F} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

- DIVERGENCE or FLUX DENSITY of a 2D vector field $\mathbf{F} = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is defined as the following scalar function

$$div\,\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

## 1.8. SUGGESTED REVIEW PROBLEMS

Instead of trying to solve them all in order (I am not sure you have enough time for this anyway, there are more than 100 of them), try to pick them at random and see how many you can solve. Initially, try not to look at what chapter they come from – this will force you to remember what concept to use just like in the test... Most answers are in the back of the book.

Another option is to focus on the parts about which you feel the weakest first. If you don't have enough time, try at least to look at the 50+ bold ones.

9.6: 23, 25, **29**, 33, **39**, 57, 59
9.9: **19**, 20

12.7: 21, 29, 35, 41
12.8: 1, **17,** 23, **41**, 49
12.9: 1, **13**, **19**, 25, 27, 29
12.PE: 69, **71**, 83, **84**, **97**
12.AE: 9, 13

13.1: 7, 9, 23, 27, **31**, 37, **39**, 49, 59, **61**
13.2: **11**, **13**, 15, 17, 25, **47**
13.3: **9**, 11, **25**, 41
13.4: 17, 21, **29**, 41, **48**
13.5: 13, 15ab, **17ab**
13.6: 7, 9, **11**, 13, 17, **19**, 21, **31**, 33, 39, 41, 49, **55, 63, 65,** 85
13.PE: 5, 9, **11**, 27, 43, **51**
13.AE: 1, **29, 33**

14.1: 9, 19, **23**, 25
14.2: **5**, **18**, 23, **25**, 27, 35, **43**, **44**
14.3: 1, 3, 5, **6**, 7, **17**, **23**, **29**, **32**, **37**
14.4: 5, 7, **9,** 18, 23, 25, **35**, **39**
14.PE: **4, 31, 37, 47a,** 54
14.AE: **9**