

1 Sample Midterm

Part I

Multiple choice – circle the correct answers!

1) The surface given by the equation $x^2 + y^2 = z$ intersects the plane $x = 0$ in:

- a) a parabola;
- b) a hyperbola;
- c) an ellipse;
- d) two straight lines.

2) The portion of the parametric curve $x(t) = \cos t - \sin t$, $y(t) = \cos t + \sin t$ traced out in the interval $0 \leq t \leq 2\pi$ has arclength:

- a) $\sqrt{2}\pi$;
- b) 2π ;
- c) $2\sqrt{2}\pi$;
- d) 4π .

3) If \mathbf{v} , \mathbf{w} are two vectors, which of the following quantities is $|\mathbf{v} + \mathbf{w}|^2$ equal to, no matter how \mathbf{v} , \mathbf{w} are chosen?

- a) $|\mathbf{v}|^2 + |\mathbf{w}|^2$;
- b) $(|\mathbf{v}| + |\mathbf{w}|)^2$;
- c) $|\mathbf{v}|^2 + |\mathbf{w}|^2 + 2\mathbf{v} \cdot \mathbf{w}$;
- d) $|\mathbf{v}|^2 + |\mathbf{w}|^2 + \mathbf{v} \cdot \mathbf{w}$.

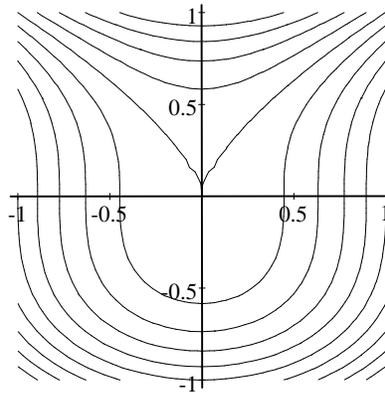
4) Three of the following equations are correct for all possible choices of vectors \mathbf{v} and \mathbf{w} , and one is not. Which one can be *wrong*?

- a) $\mathbf{v} \times (\mathbf{v} \times \mathbf{w}) = 0$;
- b) $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w}) = 0$;
- c) $|\mathbf{v} \times \mathbf{w}|^2 + (\mathbf{v} \cdot \mathbf{w})^2 = |\mathbf{v}|^2 |\mathbf{w}|^2$;
- d) $(\mathbf{v} + \mathbf{w}) \cdot (\mathbf{v} - \mathbf{w}) = |\mathbf{v}|^2 - |\mathbf{w}|^2$..

5) The quantity $w = w(x, y)$ is a function of the variables x, y , which in turn depend on the variables s, t according to the equations $x = s + t$, $y = s - t$. Which of the following identities is correct?

- a) $\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}$
- b) $\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 2 \frac{\partial w}{\partial x}$
- c) $\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = \frac{\partial w}{\partial y}$
- d) $\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 2 \frac{\partial w}{\partial y}$

6) Which one of the functions below corresponds to the following unlabeled set of level curves?



(the x -axis is horizontal)

- a) $f(x, y) = x^2 - y^3$
- b) $f(x, y) = x^3 - y^2$
- c) $f(x, y) = x^4 - y^2$
- d) $f(x, y) = y - x^2$

Part II

You should attempt all parts of all problems. Show your work!

- 1) Consider the two vectors $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$
- What is the angle between the vectors \mathbf{v} and \mathbf{w} ?
 - What is the area of the triangle that has the vectors \mathbf{v} and \mathbf{w} as sides?
 - Let Π be the plane through the point $(5, 1, 1)$ and parallel to the vectors \mathbf{v} , \mathbf{w} . Give an equation describing this plane.

2) The following problem was contributed by a Star Wars fan among the section leaders: You are a Jedi Knight. To save the Rebellion, you must destroy the Empire's Death Star by shooting your laser at its power reactor. Guided by The Force, your X-Wing fighter follows the curve (in the x - y plane) given by the position vector $\mathbf{r}(t) = t\mathbf{i} + 10e^{-t}\mathbf{j}$ - you have no control over this. When you fire your laser at time t , its beam travels in a straight line in the direction of your instantaneous velocity vector $\mathbf{r}'(t)$. Suppose you fire your laser at time $t = T$.

- Give a parametrization of the line on which your laser beam travels using the parameter s , such that $s = 0$ corresponds to $\mathbf{r}(T)$.
- At what location will the laser beam strike the x -axis?
- The Death Star's reactor is located at the point $(3, 0)$. At what time should you fire your laser?

3) Let $T(x, y) = e^{-(x^2+y^2)}$ be the temperature distribution on a plate, measured in degrees Fahrenheit. A bug is at the point $(2, 3)$ on the plate.

a) If the bug is crawling along an isothermal curve (i.e., a level curve of the temperature function), give a possible parametrization of the bug's path, tracing out the entire isothermal curve. Note: the bug need *not* be at point $(2, 3)$ at $t = 0$.

b) If the bug is instead travelling along the curve with position vector

$$\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + (4t - t^2)\mathbf{j},$$

what is $\frac{dT}{dt}$ at the moment when the bug reaches the point $(2, 3)$?

4) Recall that the equation of motion for a planet orbiting a star can be given by

$$\frac{d^2\mathbf{r}}{dt^2} = -GM\frac{\mathbf{r}}{|\mathbf{r}|^3},$$

where \mathbf{r} is the position vector of the planet, M is the mass of the star and G is a physical constant. Show that

$$E(t) = \frac{1}{2}mv^2 - G\frac{mM}{|\mathbf{r}|}$$

is a constant, where $v = |\mathbf{v}| = \left|\frac{d\mathbf{r}}{dt}\right|$ is the speed of the planet and m is its mass.