Math 21a Practice Exam 2
Give yourself 90 minutes to try this exam.

(1) The function $x^3 + x^2y + 4xy + 4y^2$ has stationary points at $x = 0$, $x = 2$, and $x = 4$. For each of these stationary points, find the value of y, and classify the stationary point as a maximum, minimum, or saddle point.

(2) Find the maximum and minimum values of the function $g(x, y) = x^2 + y^2 - 2x + 4y$ on the region whose boundary is the circle $x^2 + y^2 = 20$.

(3) Suppose that $F(x,t) = g(x - vt)$, where g is a twice-differentiable function of its one argument. Show that $\frac{\partial^2 F}{\partial t^2} = v^2 \frac{\partial^2 F}{\partial x^2}$.

(4) The equation $x + z + (y + z)^2 = 6$ defines z implicitly as a function of x and y near the point $(x, y, z) = (1, 1, 1)$. Find the partial derivatives of the function $z(x, y)$ at this point, and use them to estimate the value of z for $x = 1.2$, $y = 1.1$.

(5) Functions u and v are defined in terms of x and y by $u = xy$, $v = \ln y - \ln x$ for x and y greater than 0.

(a) Suppose that a particle is passing through a point where $x = 2$ and $y = 4$ with $\frac{dx}{dt} = 3$, and that $\frac{dv}{dt} = 0$ at this instant. Use the chain rule to determine $\frac{du}{dt}$ at this instant.

(b) Suppose that the temperature T is given by a function $f(u, v)$. Determine the first partial derivatives of T with respect to x and y in terms of x, y, and the partial derivatives of the function f.

(6) Set up and evaluate an integral in cylindrical coordinates to determine the average value of the function $x^2 + y^2$ over a cone whose height is b and whose base radius is a.
[Assume the central axis of the cone lies on the z-axis.]

(7)(a) Evaluate $\iint_D y\,dxdy$ over the region between the parabola $y = x^2$ and the line $y = x$ as an iterated integral, integrating first over y, then over x.

(b) Evaluate the same integral with the opposite order of integration.