Math 21a - Spring 2001 - Problem Set #3 and Exam Study Guide
(problems due no later than Wed, March 7 at 1:00pm)

Dates covered: Feb 26 - Mar 2
Key Topics: (Sections 2.2-2.5, 5.2, and the Basic Chain Rule) Partial derivatives. Idea of continuity and differentiability of functions of several variables. Linear approximation and tangent planes. Rate of change of a function along a parametrized curve and the Basic Chain Rule. Directional derivatives and the gradient vector field. Conservative vector fields, independence of path, and the Fundamental Theorem of Line Integrals.

The Problems:
From the Ostebee-Zorn text:

(1) 2.2/12,16

(2) 2.3/14,18,22

(3) 2.4/4,8,12

(4) 2.5/2

(5) 5.2/2,6,8

Things you may want to know for the exam on March 7:

1. Given vectors in $\mathbf{R}^2$ or $\mathbf{R}^3$, do addition, scalar multiplication, dot and cross products.

2. Manipulate vector expressions symbolically (distributive law, triple product, $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$).

3. Express lengths, angles, areas of triangles and parallelograms, and volumes of parallelepipeds and tetrahedra in terms of vectors. Identify and construct orthogonal vectors.

4. Use vectors to formulate and prove geometric and trigonometric theorems. (as in the problem sets)

5. Given sufficient data, describe a line or plane either parametrically or in terms of one or more equations involving x, y, and z.

6. Calculate the intersection (if any) of specified lines and planes. Resolve a vector into components parallel and perpendicular to a specified vector, line, or plane.

7. Calculate the distance from a point to a specified line or plane, or between two nonintersecting lines in $\mathbf{R}^3$.

8. For motion in $\mathbf{R}^2$ or $\mathbf{R}^3$ specified parametrically in terms of time, calculate and sketch position, velocity, and acceleration vectors, and find an expression for the tangent line to the path at a specified point and for the unit tangent vector.

9. Solve problems based on motion of one or two particles described parametrically; e.g. when does it cross a specified plane, when is it closest to a specified point, what is the distance of closest approach of two moving particles?

10. For motion with specified constant acceleration and specified initial velocity and position, write a parametrized expression for the path.

11. Write one or more parametrized expressions for a specified path in $\mathbf{R}^2$ or $\mathbf{R}^3$.

12. Develop an expression for the length of a parametrized path as an integral.

13. Sketch a graph of a parametrized path, and eliminate the parameter (if possible) to find an equation or equations in terms of x, y, (and z) for the path.

14. Make or identify sketches of vector fields in $\mathbf{R}^2$.

15. Calculate the line integral of a specified vector field over a specified path, inventing a parametrization if necessary.

16. Write equations for surfaces in $\mathbf{R}^3$ that are characterized geometrically by distances (spheres, cylinders, etc.), and determine their intersection with specified planes.

17. Sketch or identify contour plots of real-valued functions on $\mathbf{R}^2$. Identify plots of function graphs for such functions.

18. Calculate partial derivatives and directional derivatives of functions of two or more variables.

19. Calculate the gradient of a real-valued function of two or three variables, and state and apply the relation between gradients and level curves or surfaces.

20. Calculate the rate of change of a real-valued function along a parametrized path.

[Basic Chain Rule: $\frac{d}{dt}[f(x(t),y(t))] = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \nabla f \bullet \mathbf{v}$ for a path in $\mathbf{R}^2$;

$\frac{d}{dt}[f(x(t),y(t),z(t))] = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} = \nabla f \bullet \mathbf{v}$ for a path in $\mathbf{R}^3$.]

21. Given a surface in $\mathbf{R}^3$ described by the graph of a function $f(x, y)$, find the equation of the tangent plane at a point on the surface where f is known to be differentiable, and use it to find approximate values for this function near the point of tangency.
[Linear approximation: $f(x,y) \cong f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$]

22. Test whether a given vector field in $\mathbf{R}^2$ or $\mathbf{R}^3$ is conservative. For a conservative field, find a (potential) function of which it is the gradient.

23. Calculate the line integral of a conservative vector field using the Fundamental Theorem of Line Integrals, i.e. $\int_\gamma \nabla V \bullet d\mathbf{x} = V(\mathbf{x}(b)) - V(\mathbf{x}(a))$ for a path γ described by $\mathbf{x}(t)$ for $a \le t \le b$.

In addition to office hours and the Math Question Center, all discussion sections conducted by the Math 21a Course Assistants are open to all. Here is the schedule:

Monday, 6-7:30pm in SC209, with Nathan Moore

Monday, 7:30-9pm in SC310, with Michael Simonetti

Monday, 8-9pm in SC101b with Bryden Cais

Tuesday, 6:30-8pm SC411, with Lisa Carlivati

Tuesday, 7-8:30pm in SC216, with Anthony Varilly

Tuesday, 8:30-10pm in SC304, with Geoff Werner-Allen