## Section 2.2

12. The climber will begin to ascend. Since $\frac{\partial z}{\partial x} = -6x$, the climber will begin to ascend at a rate of 90 vertical feet per horizontal foot if the climber walks due west. (NOTE: The fact that $z_x(15, -10) = -90 < 0$ implies that moving due east leads to a decrease in the climber's elevation.)

16. (a) $\frac{\partial}{\partial T}\left(\frac{2\pi r}{T}\right) = -\frac{2\pi r}{T^2}$

    (b) $\frac{\partial F}{\partial m_1} = \frac{Gm_2}{r^2}$

    (c) $\frac{\partial}{\partial y}\sin(3x^4y + 2x^3y^5) = (3x^4 + 10x^3y^4)\cos(3x^4y + 2x^3y^5)$

    (d) $\frac{\partial}{\partial x}\left(xe^{\sqrt{xy}}\right) = \left(1 + \sqrt{xy}/2\right)e^{\sqrt{xy}}$

## Section 2.3

14. (a) $f_x(x, y) = \cos x + y$; $f_y(x, y) = 2 + x$, so $f_x(0, 0) = 1$ and $f_y(0, 0) = 2$.

    (b) The linear approximation to $f$ at $(0, 0)$ is $L(x, y) = x + 2y$.

    (c) All parts are routine, given the formulas. The two rows are have similar entries, especially at the left, because $L$ approximates $f$ closely near $(0, 0)$.

    (d) Corresponding level curves of the two functions should appear similar, especially near $(0, 0)$.

18. (a) The information given on partial derivatives implies that $L(x, y) = 0.6(x - 3) + 0.8(y - 4) + 5 = 0.6x + 0.8y$.

    (b) Use the formula above: $L(2.9, 4.1) = 0.6(-0.1) + 0.8(0.1) + 5 = 5.02$; $L(4, 5) = 0.6(1) + 0.8(1) + 5 = 6.4$.

    (c) The function $g$ can not be linear. If $g$ were linear it would agree everywhere with $L$—but we saw above that $g(4, 5) = \sqrt{41} \neq 6.4 = L(4, 5)$.

22. $(2, 1, 1)$

## Section 2.4

4. (a) The tangent plane is $z = 1$.

   (b) The tangent plane is $x + y + z = \sqrt{3}$.

   (c) The tangent plane is $6x - 2y - z = 9$.

   (d) The tangent plane is $4x + 2y - z = 5$.

8. (a) ascend at rate of 90 feet/foot

   (b) descend at rate of $85\sqrt{2}$ feet/foot

   (c) $(1, 9/8)$

12. (a) $(2, -2)$

    (b) $-2/\sqrt{13}$

## Section 2.5

2. (a) The partial derivative $g_y(0, 0) = 0$; the partial derivative $g_x(0, 0)$ does not exist.

   (b) Setting $y = mx$ in the formula gives $g(x, y) = 1/(1 + m^2)$.

   (c) The contour lines $g(x, y) = \pm 1/2$, $g(x, y) = \pm 1/5$, and $g(x, y) = \pm 1/10$ are lines of slope $\pm 1$, $\pm 2$, and $\pm 3$, respectively. *Maple* has trouble because the function is discontinuous.

   (d) If $\boldsymbol{u} = (1/\sqrt{2}, 1/\sqrt{2})$, the directional derivative $D_{\boldsymbol{u}}g(0, 0)$ does not exist, since the limit that defines it doesn't exist.

   (e) The directional derivative $D_{\boldsymbol{u}}g(0, 0)$ exists only if $\boldsymbol{u} = \boldsymbol{j}$.

   (f) The surface $z = \frac{x^2}{x^2+y^2}$ is "torn" near the origin.

2. (a) Letting $\boldsymbol{X}(t) = (\cos t, \sin t)$, for $0 \le t \le 2\pi$, gives $\int_\gamma \boldsymbol{f}(\boldsymbol{X}) \cdot d\boldsymbol{X} = 2\pi$. The answer is positive because the curve "goes with the flow."

   (b) Parametrize the circle using $\boldsymbol{X}(t) = (1 + \cos t, \sin t)$, for $0 \le t \le 2\pi$; then the line integral reduces to $\int_0^{2\pi} (1 + \cos t - \sin t)\, dt = 2\pi$. The answer is positive because, again, the curve "goes with the flow."

   (c) Parametrize the curve using $\boldsymbol{X}(t) = (1 + \cos t, \sin t)$, for $0 \le t \le \pi$; then the line integral reduces to $\int_0^{\pi} (1 + \cos t - \sin t)\, dt = \pi - 2$. The answer is positive because the curve is still oriented "with the flow."

6. In cases where the integrand is a gradient field, we use the fundamental theorem. For the other fields, we use brute force (and this parameterization $\gamma$: $x(t) = t$, $y(t) = t$, $1 \le t \le 2$).

   (a) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = 4 - 1 = 3$

   (b) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = 4 - 1 = 3$

   (c) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = \int_1^2 (-t, t) \cdot (1,1)\, dt = 0$

   (d) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = \int_1^2 (1, \sin t) \cdot (1,1)\, dt = \int_1^2 (1 + \sin t)\, dt = 1 + \cos 1 - \cos 2.$

   (e) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = 1 + \cos 2 - \cos 1.$

   (f) $\int_\gamma P\,dx + Q\,dy = h(2,2) - h(1,1) = (\ln 8)/2 - (\ln 2)/2 = \ln 2.$

8. Yes — $\boldsymbol{f}(x, y) = \nabla h(x, y)$ when $h(x, y) = \sin(x^3 + y)$.