Math 21a - Spring 2001 - Problem Set #4
(due Mon, Mar 19 or Tues, Mar 20)

Dates: Mar 9 - Mar 16
Text sections: 2.6, 2.7, 4.4, and supplement on Lagrange Multipliers.
Key Topics: Higher order derivatives & quadratic approximation, extrema of functions of several variables, unconstrained optimization, constrained optimization and the Method of Lagrange Multipliers.

(1)(a) Find a linear approximation L(x, y) for the function $f(x, y) = x^3e^y - 2xy^2$ in the vicinity of the point $(x_0, y_0) = (2,0)$. Estimate f (2.1,-0.2) using this linear approximation.
(b) Find a quadratic approximation Q(x, y) for the same function as above in the vicinity of the point $(x_0, y_0) = (2,0)$. Estimate f (2.1,-0.2) using this quadratic approximation.

(2) Ostebee-Zorn problems: 2.7/8,10. [Bonus for illustrative Mathematica graphs]

(3) Ostebee-Zorn problems: 2.7/20,26. [Bonus for illustrative Mathematica graphs]

(4) Ostebee-Zorn problems: 4.4/2,8.

(5) Lagrange Multiplier supplement problems: 8,10,12,13.

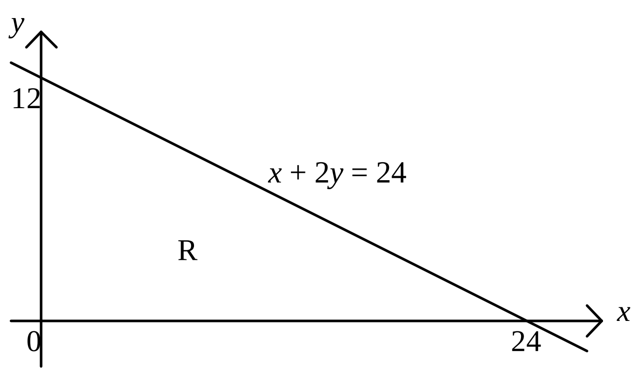
(6) Suppose that the moon is modeled by the ball where $x^2 + y^2 + z^2 \le 1$. If the temperature of a point (x, y, z) in or on the moon at a particular time is given by the function

$$T(x, y, z) = 50(1 - x^2 - y^2 - z^2) + 10(\sqrt{3}\,x + z)$$

then find:
a) The (x, y, z) coordinates of the hottest of the points on the surface of the moon.
b) The (x, y, z) coordinates of the points in or on the moon where temperature is greatest.
c) The (x, y, z) coordinates of the points in or on the moon where the temperature is least.
In all cases, make sure you justify your conclusions.

(7) Two planes are given by the equations $2x + y + z = 2$ and $x - y - 3z = 4$. Find the point on their line of intersection that is closest to the origin.

(8) Find the maximum and minimum values of the function $f(x, y) = x^2 - xy + 2y^2 - 6x - 4y$ in the triangular region R shown. Investigate all possibilities.
[Bonus credit: Produce a contour diagram for this function superimposed with the region R.]



(9) Find points with extreme temperature on the solid (filled-in) ellipsoid $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{4} \le 1$ if the temperature at any point (x, y, z) in the solid is given by $T(x, y, z) = 2x^2 + yz - 4z + 150$.