Math 21a Solutions to Problem Set #4

① Recall $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

and $Q(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + \frac{1}{2}f_{xx}(x_0,y_0)(x-x_0)^2 + f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + \frac{1}{2}f_{yy}(x_0,y_0)(y-y_0)^2$

So let's calculate some partials!

$f = x^3e^y - 2xy^2$	$f(2,0) = 8$
$f_x = 3x^2e^y - 2y^2$	$f_x(2,0) = 12$
$f_y = x^3e^y - 4xy$	$f_y(2,0) = 8$
$f_{xx} = 6xe^y$	$f_{xx}(2,0) = 12$
$f_{xy} = f_{yx} = 3x^2e^y - 4y$	$f_{xy}(2,0) = 12$
$f_{yy} = x^3e^y - 4x$	$f_{yy}(2,0) = 0$

So

ⓐ $\boxed{L(x,y) = 8 + 12(x-2) + 8y \quad L(2.1,-0.2) = 7.6}$

ⓑ $\boxed{Q(x,y) = 8 + 12(x-2) + 8y + \frac{1}{2}\cdot 12(x-2)^2 + 12(x-2)y \quad Q(2.1,-0.2) = 7.42}$

Actual value $f(2.1,-0.2) \approx 7.414$

So $Q(2.1,-0.2)$ is a better approximation than $L(2.1,-0.2)$ — does this make sense to you? (Think about 1-variable analog, Taylor approximations.)

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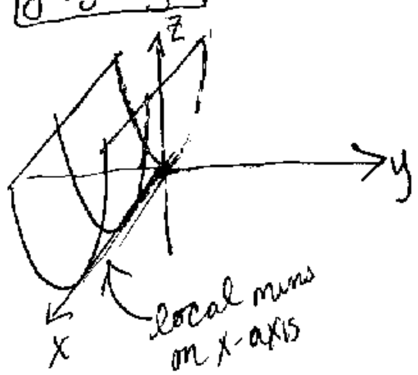
② O-Z 2.7/8:

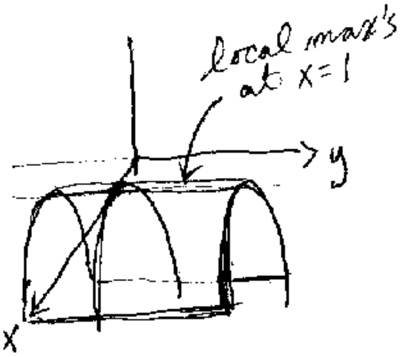
There are infinitely many solutions that satisfy these conditions. For simplicity, recall that the vertex of a parabola will be a max or min depending on what direction it opens. Can we find solutions using parabolic cylinders or paraboloids?

One possible set of solutions:

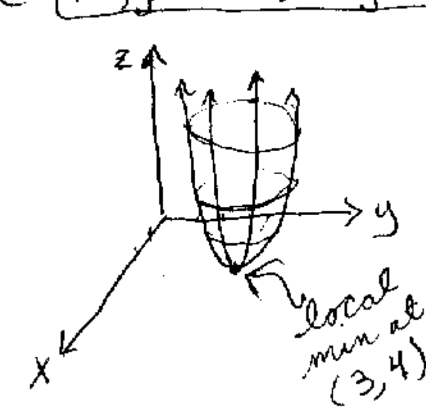
ⓐ $\boxed{g(x,y) = y^2}$



ⓑ $\boxed{h(x,y) = -(x-1)^2}$



ⓒ $\boxed{k(x,y) = (x-3)^2 + (y-4)^2}$



(2) O-Z 2.7/8 (For the More "Mathematica"-ly minded...)

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In[42]:= a3D = Plot3D[y^2, {x, -5, 5}, {y, -5, 5}]
         acontour = ContourPlot[y^2, {x, -5, 5}, {y, -5, 5}]
         b3D = Plot3D[ - (x - 1) ^2, {x, -4, 6}, {y, -5, 5}]
         bcontour = ContourPlot[- (x - 1) ^2, {x, -4, 6}, {y, -5, 5}]
         c3D = Plot3D[ (x - 3) ^2 + (y - 4) ^2, {x, -2, 8}, {y, -1, 9}]
         ccontour = ContourPlot[ (x - 3) ^2 + (y - 4) ^2, {x, -2, 8}, {y, -1, 9}]
         Show[GraphicsArray[{{a3D, acontour}, {b3D, bcontour}, {c3D, ccontour}}]];

Out[47]= - ContourGraphics -
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