(2) OZ 2.7/10:

$f = x^2 + y^2 - x^2 y/2$

So $\nabla f = (2x - xy,\ 2y - x^2/2)$

Stationary points are where $\nabla f = 0$.

So $2x - xy = 0$ $\qquad 2y - x^2/2 = 0$

$x(2-y) = 0 \qquad 4y = x^2$

$\left.x = 0 \text{ or } y = 2\right\} \longrightarrow$ If $x = 0$,

$4y = 0 \rightarrow y = 0$

So $(0,0)$ is stationary

If $y = 2$,

$8 = x^2 \rightarrow x = \pm 2\sqrt{2}$

So $(\pm 2\sqrt{2}, 2)$ are stationary

Now let's calculate H_f:

$$H_f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 2-y & -x \\ -x & 2 \end{bmatrix}$$

$$H_f(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \quad \det(H_f(0,0)) = 4 - 0 > 0$$

So $(0,0)$ is a max or min. Since $f_{xx} > 0$, by second derivative test, $\boxed{(0,0) \text{ is a local minimum.}}$

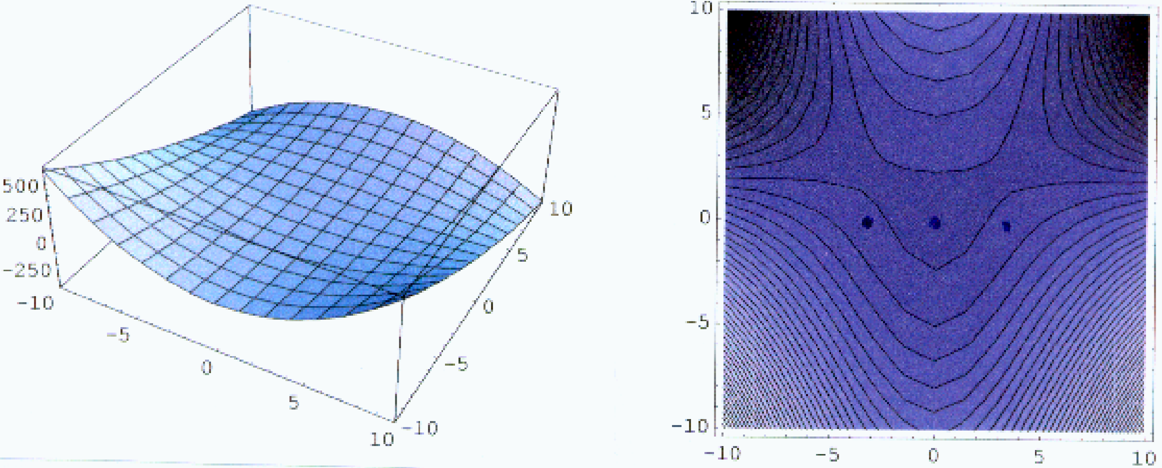
$$H_f(2\sqrt{2}, 0) = \begin{bmatrix} 2 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{bmatrix}. \quad \det(H_f(2\sqrt{2}, 0)) = 4 - 8 < 0$$

So $\boxed{(2\sqrt{2}, 0) \text{ is a saddle point}}$

$$H_f(-2\sqrt{2}, 0) = \begin{bmatrix} 2 & 2\sqrt{2} \\ 2\sqrt{2} & 2 \end{bmatrix}. \quad \det(H_f(-2\sqrt{2}, 0)) = 4 - 8 < 0$$

So $\boxed{(-2\sqrt{2}, 0) \text{ is a saddle point}}$

```
In[66]:= Show[GraphicsArray[{Plot3D[x^2 + y^2 - x^2 y / 2, {x, -10, 10}, {y, -10, 10}],
           ContourPlot[x^2 + y^2 - x^2 y / 2, {x, -10, 10}, {y, -10, 10}, Contours → 53]}]];
```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-5
-10
-10
-5
0
5
10

③ O-Z 2.7/20:

$$f = x^3 + y^3 + 3x^2 - 3y^2 - 8$$

$$\nabla f = (3x^2 + 6x,\ 3y^2 - 6y)$$

Stationary points are where $\nabla f = 0$,

So $3x^2 + 6x = 0$ $\qquad$ $3y^2 - 6y = 0$

$3x(x+2) = 0$ $\qquad$ $3y(y-2) = 0$

$x = 0$ or $x = -2$ $\qquad$ $y = 0$ or $y = 2$

Since these conditions are independent, each x value may be paired with each y-value. Hence, there are 4 stationary points:

$(0,0)$, $(0,2)$, $(-2,0)$, $(-2,0)$

$$H_f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 6x+6 & 0 \\ 0 & 6y-6 \end{bmatrix}$$

$H_f(0,0) = \begin{bmatrix} 6 & 0 \\ 0 & -6 \end{bmatrix}$, $\det(H_f(0,0)) = -36 - 0 < 0$

So $\boxed{(0,0) \text{ is saddle point}}$

$H_f(0,2) = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, $\det(H_f(0,2)) = 36 - 0 > 0$

So $(0,2)$ is a max/min. Since $f_{xx} = 6 > 0$,

$\boxed{(0,2) \text{ is a local min}}$

$H_f(-2,0) = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}$, $\det(H_f(-2,0)) = 36 - 0 > 0$

and since $f_{xx} = -6 < 0$,

$\boxed{(-2,0) \text{ is a local max}}$

$H_f(-2,2) = \begin{bmatrix} -6 & 0 \\ 0 & 6 \end{bmatrix}$, $\det(H_f(-2,2)) = -36 - 0 < 0$

So $\boxed{(-2,2) \text{ is a saddle point}}$

```
Show[GraphicsArray[
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    x^3 + y^3 + 3x^2 - 3y^2 - 8, {x, -3, 1}, {y, -1, 3}, Contours → 10]}]];
```

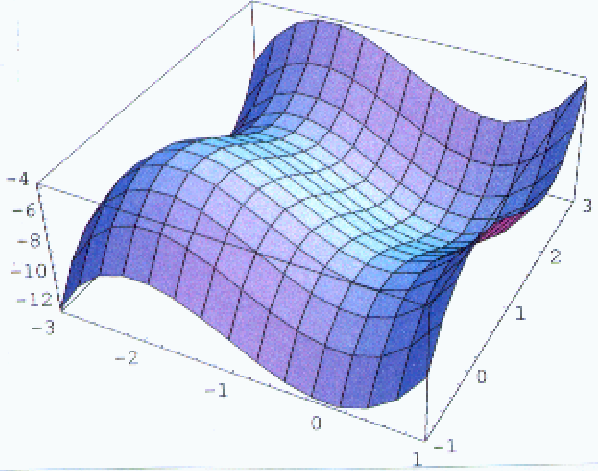
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-10
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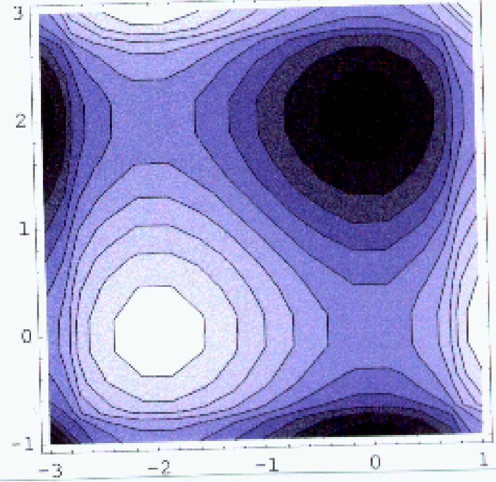
-3
-2
-1
0
1
-1
0
1
2
3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0
-1
-3
-2
-1
0
1