

Math 21a - Spring 2001 - Problem Set #6 and Exam Study Guide
(problems due no later than Fri, April 13)

Dates covered: Apr 2-6

Text sections: 3.1 - 3.4

Key Topics: Integrals over regions in \mathbf{R}^2 in Cartesian and polar coordinates; integrals over regions in \mathbf{R}^3 in Cartesian, cylindrical, and spherical coordinates; averaging, centroids, and center of mass.

(1) Ostebee-Zorn problem 3.2/12

(2) Ostebee-Zorn problem 3.3/6.

(3) Ostebee-Zorn problem 3.4/10.

(4) Ostebee-Zorn problem 3.4/12.

(5) Calculate the integral $\iint_R 2e^{x^2} dA$ where R is the triangle where $0 \leq y \leq 1$ and $y \leq x \leq 1$.

Thus, the vertices of R are $(0, 0)$, $(1, 0)$ and $(1, 1)$ in the xy plane.

(6) Integrate $f(x, y) = \frac{x}{y}$ over the region in the first quadrant cut out by the four lines whose equations are

$y = x$, $y = 2x$, $x = 1$, and $x = 2$.

(7) Find the integral of $2/(1 + (x^2 + y^2)^{1/2})$ on the region where $-1 \leq x \leq 0$ and $-(1 - x^2)^{1/2} \leq y \leq 0$.

(8) Find the area enclosed by 1 leaf of the rose $r = 12 \cos 3\theta$.

(9) Let V be the volume inside the cylinder where $0 \leq z \leq 10$ and $x^2 + y^2 \leq 1$.

The density function for the interior of this cylinder is $\sigma(x, y, z) = (100 - z^2)(1 - x^2 - y^2)$.

(a) Compute the total mass in the cylinder.

(b) Compute its center of mass.

[Note: The density function was changed from what was originally posted.]

(10) The region that lies inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 1$ is the base of a solid, right cylinder whose height is given by $z = r \cos \theta$. Find the cylinder's volume.

Things you may want to know for the second hour exam

24. Given a function of two or three variables, locate all its stationary points, or determine whether or not a specified point is a stationary point.
25. For a function of two variables, construct an approximating function near a specified point (x_0, y_0) that includes both linear and quadratic terms in $(x - x_0)$ and $(y - y_0)$.
26. Write down the Hessian matrix for a function of two variables at a given stationary point and use it to determine whether the stationary point is a minimum, a maximum, or a saddle point. Identify cases where the Hessian cannot answer this question.
27. Sketch or identify level curves of a function of two variables in the vicinity of a stationary point.
28. Use the method of Lagrange multipliers to find the stationary points of a function $f(x, y)$ of two variables subject to a constraint $g(x, y) = \text{constant}$. Sketch or identify level curves and gradient vectors for f and g in the vicinity of one of these stationary points.
29. Use the method of Lagrange multipliers to find the stationary points of a function $f(x, y, z)$ of three variables subject to a constraint $g(x, y, z) = \text{constant}$.
30. Formulate and solve optimization problems that involve minimizing a sum of squares.
31. Given a region of the plane that includes its boundary, enumerate all the points that are candidates for the location of the maximum or minimum value of the function on that region.
32. Find the maximum and minimum values of a specified function on a region of the plane that is bounded by one, two, or three lines or curves.
33. Given a function $f: \mathbf{R}^m \rightarrow \mathbf{R}^n$, express the linear approximation to the function at a specified point using a matrix of partial derivatives.
34. Given functions $g: \mathbf{R}^m \rightarrow \mathbf{R}^n$ and $f: \mathbf{R}^n \rightarrow \mathbf{R}^p$, state the chain rule for the derivative of $f \circ g$, and express it in terms of multiplication of matrices of partial derivatives.
35. Given a function $s = f(u, v)$ or $s = f(u, v, w)$ whose arguments u, v, w are specified functions of x, y , and perhaps z , use the chain rule to derive or verify relationships among the partial derivatives of s with respect to x, y , and z .
36. Given functions that express Cartesian coordinates x and y in terms of other coordinates u and v , and a function $z = f(x, y)$, use the chain rule to express partial derivatives of z with respect to u and v in terms of partial derivatives of f with respect to x and y .
37. Use the chain rule to solve “related-rate problems,” i.e. apply the chain rule in situations when the numerical values of the derivative of $f \circ g$ must be determined in the case when you do not know the functional form of g , but only the numerical values of the partial derivatives of g at the appropriate point.
38. Given a curve in the plane specified by $f(x, y) = \text{constant}$, use implicit differentiation to find a formula for the derivative of the function that specifies y in terms of x near a specified point on the curve, and use the value of this derivative to determine a tangent line or to do a linear approximation near the point.
39. Given a surface specified by $f(x, y, z) = \text{constant}$, use implicit differentiation to find formulas for the partial derivatives of the function that specifies z in terms of x and y near a specified point on the surface, and use the values of these derivatives to determine a tangent plane or to do a linear approximation near the point.
40. Write down or identify a Riemann sum whose limit is the double or triple integral of a specified function over a specified region.
41. Given a region in the plane that can be divided into strips bounded by function graphs, express a double integral over the region as an iterated integral in Cartesian coordinates, and evaluate the iterated integral by using antiderivatives, if possible.
42. Given a region in \mathbf{R}^3 that is bounded by planes (not necessarily all perpendicular to the coordinate axes), express a triple integral over the region as an iterated integral in Cartesian coordinates, and evaluate the iterated integral by using antiderivatives, if possible.
43. Given an iterated double or triple integral, identify the domain of integration and express the integral as an iterated integral with a different order of integration.
44. Given a region in the plane that is bounded by circular arcs, radial lines, and graphs of functions expressing r in terms of θ , express a double integral over the region as an iterated integral in polar coordinates, and evaluate the iterated integral by using antiderivatives, if possible.
45. Given an iterated double or triple integral in one coordinate system, identify the domain of integration and express the integral in terms of a different coordinate system.
46. Given a region in \mathbf{R}^3 whose boundary consists of one or two planes perpendicular to the z -axis plus part of a cylinder, cone or sphere symmetrical about the z -axis, express a triple integral over the region as an iterated integral in cylindrical coordinates, and evaluate the iterated integral by using antiderivatives, if possible.