Math 21a - Spring 2001 - Problem Set #8

Dates covered: April 16-25
Text sections: 5.4, 5.5, 5.6, 5.7, and supplements.
Key topics: Parametrized surfaces in $\mathbf{R}^3$; Surface integrals, surface area; Flux of a vector field through a surface; Green's Theorem; Divergence Theorem.

(1) Ostebee-Zorn problems 5.5/4

(2) Ostebee-Zorn problems 5.6/2,4

(3) Take a parameterized curve C: $(f(u), g(u))$ where $g(u) > 0$ in the xy-plane and revolve it about the x-axis in space to create a surface (called a "surface of revolution").
Show that $\mathbf{X}(u, v) = (f(u), g(u) \cos v, g(u) \sin v)$ parameterizes the surface where $0 \le v \le 2\pi$. Sketch the surface of revolution in the case where $f(u) = u$ and $g(u) = 1$, and also in the case where $f(u) = \cos u$ and $g(u) = \sin u + 2$. [Feel free to use Mathematica to make your sketches.]

(4) Find a parameterization for the surface obtained by revolving the curve $y = e^x$ about the x-axis.

(5) Find a parameterization for the surface where $x - y^2 + z^4 y^4 = 0$.

(6) The charge density on the surface of the infinitely long cylinder where $x^2 + y^2 \le 1$ is given by $\sigma(x, y, z) = e^{-|z|}$. Compute the total charge on the cylinder.

(7) The charge density on the sphere where $x^2 + y^2 + z^2 = 1$ is given by $\sigma(x, y, z) = z^2$. Compute the total charge on the sphere.

(8) Water is flowing down a vertical cylindrical pipe of radius 2 inches. The velocity vector field of the water is given by $\mathbf{v} = (r^2 - 4)\mathbf{k}$ where r is the distance in inches from the center of the pipe. How much water flows out of the bottom of the pipe in 3 seconds?

(9) Calculate the work integral $\int_\gamma \mathbf{F} \cdot \mathbf{dx}$ for the curve γ shown, a semicircle from the point $(4, 1)$ to the point $(6, 3)$ where $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (3x - 2y)\mathbf{j}$.
(*Hint*: It's possible to find the value of this integral without parametrizing the semicircle.)
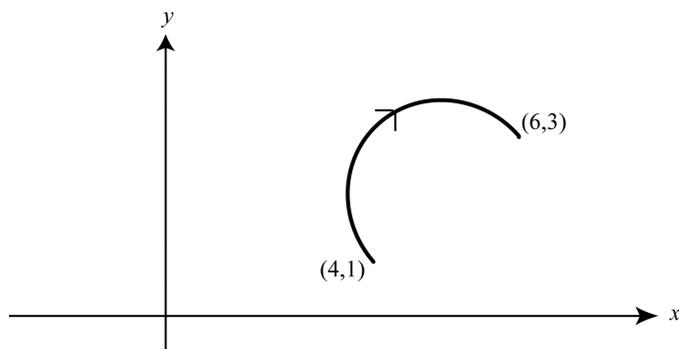
(10) Compute the flux of the vector field $\mathbf{F}(x, y, z) = (e^{y^2 + z^2}, y^2 + z^2, e^{x^2 + y^2})$ across a portion of the cone with equation $4(x^2 + y^2) = 9z^2$ lying between $z = 0$ and $z = 2$ oriented with a downward normal (i.e. with a negative z-component).

(11) Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (y^2 x, y^2 x, y^2 x)$. Let D be the portion of the solid ball $x^2 + y^2 + z^2 \le 9$ which lies in the first octant (i.e. $x \ge 0$, $y \ge 0$, $z \ge 0$, $x^2 + y^2 + z^2 \le 9$). Set up, **but do not evaluate**, a triple integral <u>in spherical coordinates</u> which gives the flux of $\mathbf{F}$ out through the boundary of the region D.