

Math 21a -- Spring 2001 -- Problem Set #8
(Biochem version)

Text Sections: Rosner chapters 3,4.

Key topics: Probability of an event; Discrete random variables; Expected value and variance; Binomial distribution.

- (1) A discrete random variable \mathbf{X} has expected value $E(\mathbf{X})=3$ and variance $Var(\mathbf{X})=5$. What are expected values and variances for the following random variables: $\mathbf{X} - 1$? $2\mathbf{X}$?
- (2) For the variable \mathbf{X} from problem (1), find $E(\mathbf{X}^2 - 1)$.
- (3) A string of 4 symbols is transmitted via a modem, and the probability of an error in each of the symbols is 10%.
- (a) \mathbf{A} is the event {At least 2 errors occurred in the transmission}. Find the probability of \mathbf{A} .
- (b) \mathbf{B} is the event {At most 2 errors occurred in the transmission}. Find the conditional probability $P(\mathbf{B}|\mathbf{A})$ of \mathbf{B} given \mathbf{A} .
- (c) Are \mathbf{B} and \mathbf{A} independent? Explain.
- (4) Rosner, Problems 4.5:
Suppose we wish to check the accuracy of self-reported diagnoses of angina by getting further medical records on a subset of the cases. If we have 50 reported cases of angina, and we wish to select 5 for further review, then how many ways can we select these cases if the order of selection matters?
- (5) Rosner, Problem 4.6: Answer Problem (4) if the order of selection does matter.
- (6) The random variables \mathbf{X} and \mathbf{Y} represent the number of bacterial colonies in two agar plates. The plates have different areas, so the probability distributions of \mathbf{X} and \mathbf{Y} are not equal: \mathbf{X} has Poisson distribution (that is, $P(\mathbf{X} = k)=e^{-\mu}\mu^k/k!$) with $\mu = 1$, while the Poisson distribution of \mathbf{Y} has $\mu = 2$.
- (a) What is the probability that the total number of colonies ($\mathbf{X} + \mathbf{Y}$) equals zero? (Hint: Do you think the events $\{\mathbf{X} = 0\}$ and $\{\mathbf{Y} = 0\}$ are independent?)
- (b) What is $P(\mathbf{X} + \mathbf{Y})=1$? (That is, the probability that the total number of colonies is 1)
- (c) (Extra credit) Find $P(\mathbf{X} + \mathbf{Y})=8$.

- (7) Calculate the work integral $\int_{\gamma} \mathbf{F} \cdot d\mathbf{x}$ for the curve γ shown, a semicircle from the point (4, 1) to the point (6, 3) where $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (3x - 2y)\mathbf{j}$.
(Hint: It's possible to find the value of this integral without parametrizing the semicircle.)

