

**Math 21a – Spring 2001 – Problem Set #9 (Biochem section)**

**Text Sections:** Rosner Chapters 4,5.

**Key topics:** 95% rule, continuous random variables: expected value and variance, normal distribution.

**Due date:** Wed, May 16.

- (1) (Rosner, 4.32) Suppose that the prevalence of hypertension among 50-59-years-olds in the general population is 18%. Suppose we identify sibships of size 3 in a community where all members of the sibship are 50-59 years old. What is the probability that 0, 1, 2, or 3 hypertensives will be identified in such sibships if the hypertensive status of 2 siblings in the same family are independent events?
- (2) In the assumptions of the previous problem, consider the sample of 25 sibships. Let the random variable  $\mathbf{X}$  be the number of sibships in this sample having at least 2 affected siblings.
- (a) What is the probability distribution of  $\mathbf{X}$ ? Expected value of  $\mathbf{X}$ ? Standard deviation of  $\mathbf{X}$ ?
- (b) (Rosner, 4.33) Suppose that among 25 sibships of this type, 5 have at least 2 affected siblings. Are these data consistent with the independence assumption in Problem (1)?
- (3) (Final Exam, Spring 2000) Let  $\mathbf{X}$  be a continuous random variable with the following probability density function:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ x/2 & 0 < x \leq 2 \\ 0 & 2 < x \end{cases}$$

- (a) Sketch  $f(x)$  (That is, its graph).
- (b) Calculate the cumulative distribution function of  $\mathbf{X}$  (By definition, the value of cumulative distribution function at a point  $x$  equals  $P(\mathbf{X} < x)$ ).
- (c) Calculate the expected value  $E(\mathbf{X})$  of  $\mathbf{X}$ .
- (d) Calculate the variance  $Var(\mathbf{X})$  of  $\mathbf{X}$ .
- (4) A dart hits the dartboard at  $(\mathbf{X}, \mathbf{Y})$ , where  $\mathbf{X}$  and  $\mathbf{Y}$  are (continuous) random variables. Suppose both  $\mathbf{X}$  and  $\mathbf{Y}$  have expected value 0 and standard deviation 2.
- (a) What is the probability  $P(|\mathbf{X}| < 3)$ ? (You may assume  $\mathbf{X}$  and  $\mathbf{Y}$  have normal distribution. Also,
- $$\frac{1}{\sqrt{2\pi}} \int_0^{3/2} \exp\left(-\frac{x^2}{2}\right) dx \approx 0.43.$$
- (b) The target is the  $6 \times 6$  square centered at the origin. What is the probability of hitting the target? (Hint: it is natural to assume  $\mathbf{X}$  and  $\mathbf{Y}$  are independent).
- (c) (Extra credit) What is the probability of hitting the circle of radius 6 centered at the origin?
- (5) Suppose the weight of a twelve-year-old male in the United States is a random variable  $\mathbf{X}$  with normal distribution. Consider the random sample of  $n = 25$  twelve-year-old males. Let  $\mathbf{Y}$  be the average weight of such a sample (in pounds). The probability density of  $\mathbf{Y}$  is approximated by
- $$f(x) = \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x-1000)^2}{32}\right).$$
- (a) What is the expected value and standard deviation of  $\mathbf{Y}$ ? (Hint: you do not have to compute any integrals to find this).
- (b) What is the expected value and standard deviation of  $\mathbf{X}$ ? What is the probability density of  $\mathbf{X}$ ? (Look up Section 5.6 in the text if you do not see how to do this).