Mathematics 21a - Spring 2001

Yet More Things You Should Know How to Do

Part 1 of this list (1-23) may be found in Problem Set #3.
Part 2 of this list (24-46) may be found in Problem Set #6.
Students in the BioChem section should note that topics after Green's Theorem differ from those presented here.

47. Given a region in $\mathbf{R}^3$ whose boundary consists of portions of one or two spheres, planes that contain the *z*-axis or that are perpendicular to the *z*-axis, and cones whose vertex is the origin and whose axis is the z-axis, express a triple integral over the region as an iterated integral in spherical coordinates, and evaluate the iterated integral by using antiderivatives, if possible.

48. Given a new coordinate system in the plane with coordinates u and v and functions $x(u, v)$ and $y(u, v)$ that express the usual Cartesian coordinates in terms of the new coordinates, along with a region of integration whose boundary can easily be expressed in terms of u and v, convert a double integral over the region into an iterated integral over u and v.

49. Given a new coordinate system in $\mathbf{R}^3$ with coordinates u, v, and w and functions $x(u, v, w)$, $y(u, v, w)$, and $z(u, v, w)$ that express the usual Cartesian coordinates in terms of the new coordinates, along with a region of integration whose boundary can easily be expressed in terms of u, v, and w, convert a triple integral over the region into an iterated integral over u, v, and w.

50. Given the line integral of a vector field over the boundary of a region in the plane, use Green's theorem to convert it to a double integral over the region.

51. Given a double integral of a function over a region in the plane, invent a vector field whose integral around the boundary of the region has the same value as the double integral.

52. Use Green's theorem for efficient calculation of area, center of mass, or moment of inertia for a region in the plane whose boundary consists of a small number of easily-parametrized curves.

53. For a region R in the plane whose boundary consists simply of portions of the lines $x = a$ and $x = b$, plus function graphs $y = f(x)$ and $y = g(x)$ over the interval $a \le x \le b$, convert the line integral of a vector field $Q(x, y)\mathbf{i}$ around the boundary of R to an integral over x, and use the Fundamental Theorem of Calculus to convert this to a double integral over the region. Show that this conversion is equivalent to a special case of Green's theorem. Equivalently, do the same with the roles of x and y reversed.

54. Given two parameters u and v, intervals for u and v, and functions that express x, y, and z in terms of u and v, identify, describe, or sketch the surface in $\mathbf{R}^3$ that is specified by this parametrization. At a given point on the surface, find two independent vectors that are tangent to the surface and one that is normal to the surface.

55. Given a surface that is a portion of a plane, sphere, cylinder, or cone in $\mathbf{R}^3$, invent a parametrization for it, choosing parameters u and v so that the vector $\mathbf{X}_u \times \mathbf{X}_v$ points in a specified direction normal to the surface.

56. Given a curve that is easily parametrized, set up and evaluate an integral to determine the length of the curve, the integral of a density function over the curve, or the average value of a function over the curve.

57. Given a surface that is easily parametrized, set up and evaluate a double integral to determine the area of the surface, the integral of a density function over the surface, or the average value of a function over the surface.

58. Given a surface that is easily parametrized, set up and evaluate an integral to determine the flux of a specified vector field through the surface.

59. Given a closed surface that consists of a small number of pieces each of which is easily parametrized (for example, a cylinder plus its top and bottom, or a cone plus its top, or a hemisphere plus a disk in its equatorial plane), evaluate the flux of a specified vector field out through this closed surface by forming the sum of appropriate flux integrals.

60. Given a vector field expressed in Cartesian coordinates, calculate the divergence or curl of the field.

61. Given a sketch of a vector field whose *z* component is zero and whose *x* and *y* components are independent of *z*, determine whether the curl of the field is or is not zero and whether the divergence is or is not zero.

62. Given the flux integral of a vector field over the boundary of a region in $\mathbf{R}^3$, use the divergence theorem to convert it to a triple integral over the region.

63. Given a triple integral of a function over a region in $\mathbf{R}^3$, invent a vector field whose flux integral over the boundary of the region has the same value as the double integral.

64. Given the line integral of a vector field around the boundary of a surface in $\mathbf{R}^3$, use Stokes' theorem to convert it to a flux integral over the surface.

65. Given a flux integral over a surface in $\mathbf{R}^3$, invent a vector field whose line integral around the boundary of the surface has (by virtue of Stokes' theorem) the same value as the flux integral.

66. By using the formulas for gradient, curl, and divergence in Cartesian coordinates, invent or verify generalizations of the product rule for differentiation.

67. Convert statements about line integrals, flux integrals, or volume integrals into equivalent statements about gradient, curl, or divergence.

68. Given an equation expressed in terms of gradient, curl, or divergence, convert it to an equivalent equation involving line integrals, flux integrals, or volume integrals.

69. Given a vector field in $\mathbf{R}^3$, some of whose Cartesian components are specified, invent functions for the other components that give the field a specified divergence or a specified curl.

70. By applying the divergence theorem or Stokes' theorem to products of vector fields, scalar fields, and/or constant vectors, derive other general results that relate line, flux, and volume integrals.