Math 21a Practice Hourly 1
(Fall 2000)

What follows is a model of the first hourly exam (it is more or less the exam given last fall in this course). The upcoming exam on Wednesday, October 18 will be roughly similar to this one in length and difficulty. To study for the upcoming exam, first work the suggested problems for the Chapters 1-2.1, and Appendix A (but not Appendix C). The exam will cover solely material from this portion of the text book. The list of suggested problems is posted on the course website. After you have worked the suggested problems and checked your answers with those in the book, try taking this exam as practice for the real thing. In this regard, note that you will have two hours for the real thing, but we hope that most people finish in well under the two hour limit. The answers to this practice hourly are provided at the website.

By the way, in the real exam, each problem will be printed on a separate page and you will be asked to provide your answers on that page and to use the back of that page if you need more space to show your work.

Remember that the exam on *Wednesday, October 18* is from *7-9pm* in *Lecture Halls C and D of the Science Center*. Please come a few minutes early as the exam will start promptly at 7pm.

1) __ 2) __ 3) __ 4) __ 5) __ 6) __ 7) __ : Total ______

Name: ____

Circle the name of your Section TA:

Allcock • Chen • Karigiannis • Knill • Liu • Rasmussen • Rogers • Taubes • Winter • Winters

Instructions:

- Print your name in the line above and circle the name of your section TF.
- Answer each of the questions below in the space provided. If more space is needed, use the back of the facing page.
- Please write neatly. Answers which are deemed illegible by the grader will not receive credit.
- No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students. Use only your brain and a pencil.
- You have 2 hours to complete your work.
- Problems 1-6 count 8 points each, while Problem 7 counts for 2 points. In budgeting your time, note that you might find Problem 6 to be quite difficult.

In agreeing to take this exam, you are implicitly accepting Harvard University's honor code.

1. Let $\mathbf{v} = (2, 1, 4)$ and $\mathbf{w} = \frac{1}{3}(1, 2, 2)$.

a) Find the lengths of $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{v} - 3\mathbf{w}$.

b) Find the scalar projection of $\mathbf{v}$ in the direction of $\mathbf{w}$.

c) Find $\mathbf{v} \times \mathbf{w}$.

d) Find numerical values for the constants b and c so that both $\mathbf{v}$ and $\mathbf{w}$ are tangent to the plane where $bx + cy + z = 0$.

2. Let $\prod$ denote the plane where $2x - 2y + z = 3$.

a) Find three points in $\prod$ which do not all lie on the same line.

b) Find a non-zero vector which is perpendicular to $\prod$.

c) Find the distance from $\prod$ to the origin.

d) Write an equation for a line which lies entirely in $\prod$.

3. A particle moving in space has position at time t given by $(3\sin(t^2), 3\cos(t^2), 4t^2)$.

a) Find the coordinates of the particle at $t = \sqrt{\pi}$.

b) Find the velocity vector of the particle at $t = \sqrt{\pi}$.

c) Find a parametric equation for the line tangent to the trajectory at $t = \sqrt{\pi}$. (By definition, this is the line through the point where the particle is at $t = \sqrt{\pi}$ whose direction is that of the particle's velocity vector at $t = \sqrt{\pi}$.)

d) Find the distance traveled by the particle between $t = 0$ and $t = \sqrt{\pi}$.

4. Let $\mathbf{v} = (5, 3, 1)$ and $\mathbf{p} = (1, 0, 1)$. It turns out that the end points of the vectors $\mathbf{r}$ which obey $(\mathbf{r} - \mathbf{p}) \times \mathbf{v} = 0$ all lie on the same line, L.

   a) Find a point on L and a tangent vector to L.

   b) Write a parametric equation for L.

   c) Find a point on L with distance 5 from the origin.

   d) Find the distance from L to the origin.

5. Let **v** = (5, 3, 1) and let L denote the line through the origin with tangent vector **v**.

a) Find an equation for some plane through the origin which contains L.

b) Let **w** = (-10, b, 2). Find a value for b which makes **w** perpendicular to **v**.

c) Let **r** = (-10, b, c). Find values for b and c which makes **v** and **r** parallel.

d) Suppose that **u** = (1, 3, -2) and **s** = (-30, -18, e). Find a value for e which makes **u** × **s** perpendicular to **v**.

6. All of the questions concern a non-stationary particle in space whose position at time t is the head of a vector $\mathbf{r}(t)$ based at the origin. For Question 6a only, assume that $\mathbf{r} \bullet \frac{d}{dt}\mathbf{r} = 0$ at all times.

   a) Answer true or false: The particle must move on the surface of a sphere. Explain your answer in a sentence

For the Questions 6b-d, use $\mathbf{k}$ to denote the vector (0, 0, 1) and suppose, instead, that $\mathbf{k} \times \mathbf{r}(t)$ is orthogonal at all times to $\mathbf{k} \times \frac{d}{dt}\mathbf{r}$.

   b) Answer true or false: The particle must move on the surface of a sphere. Explain your answer in a sentence.
   c) Answer true or false: The particle might be moving on a straight line. If you answer true, give an equation for the line. If you answer false, explain in a sentence your reasoning.
   d) Answer true or false: The particle must move on the surface of a cylinder with circular cross section. Explain your answer in a sentence.

6. Let **u** = (3927, 42, -999) and **v** = (195, 735, 1115). What is (**u**/3 - **v**/5)•(**u** × **v**)?