

Mathematics 21a - Multivariable Calculus - Spring 2001

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Office hours are tentatively Mondays and Wednesdays, noon to 1:30pm.

Prerequisites: Math 1b with a satisfactory grade, or AB-BC score of at least 4, or scores of at least (20, 8, 4) on the Harvard Math Placement Test (HMPT), or permission from the Course Head.

Textbooks: All of the sections require Multivariable Calculus by Ostebee and Zorn, published by Saunders College Publishing. Additional supplements will be made available on the Math 21a website. The BioChem sections will also require an additional biostatistics text by Rosner to be announced later in the course.

Subject matter: There are two distinct types of sections of Math 21a from which to choose: Regular/Physics and BioChem. All sections cover at least the following topics: Functions of several variables, differentiation and integration of functions of several variables, parametric curves and surfaces, optimization, vector fields, linear approximations and topics in differential equations.

The Regular/Physics sections will also cover line and surface integrals, Green's Theorem, the Divergence Theorem, and Stokes' Theorem. If there is sufficient demand for a separate Physics section, that section will use more examples drawn from physics to illustrate some of the covered topics. The assigned homework for that section may also differ from the other sections.

The BioChem section will also cover various introductory topics in statistics and probability. There will be no specific references to either biology or chemistry in this section, so the material will be accessible to all.

Detailed syllabi for the various types of sections are provided elsewhere on this website.

Which kind of section should you choose?

If you contemplate being a physics concentrator, you would benefit by being in a Regular/Physics section. If you plan to take either the Physics 15-16 or Physics 11 sequence, you should enroll in a Regular/Physics section.

If you are planning to concentrate in BioChemical Sciences, you are strongly urged by that department to enroll in the BioChem section. Anyone who will not, at some point, enroll in Physics 11 or 15-16 might consider a BioChem section as well.

Classes and problem sessions: This course is taught entirely in sections which meet three hours per week. Each section will also have a weekly 1-hour problem session conducted by the Course Assistant (CA) assigned to that section. The meeting times for the problem sessions will be arranged during the first week of classes. You are welcome to attend the problem session of any section if it fits in better with your schedule. The schedule of all problem sessions will be posted on the Calculus Office bulletin board outside of Science Center 308.

Homework: A weekly problem set will be assigned on Wednesday of each week and will be due the following Wednesday in class. Additional daily homework assignments will be given each class but need not be submitted for grading. You are strongly encouraged to discuss the homework and work on it with your fellow students. However, solutions to weekly problem sets must be written up independently by you. (This is simply a matter of professional ethics.)

Several of the weekly problem sets will require the use of mathematical software such as Mathematica, Maple, or Matlab to produce graphs and more.

Answers to the homework problems will be made available at the Math 21a course website and selected problems will be discussed in the problem sessions.

Exams: There are two course-wide midterms and a final. The midterms will take place as follows:

Midterm Exam 1: Wednesday, March 7, 4:00 to 5:30pm in Science Center Hall C

Midterm Exam 2: Wednesday, April 11, 4:00 to 5:30pm in Science Center Hall C

Please note that these exams take place in the afternoon. If you potentially have a conflict with a lab or extracurricular activity, please make arrangements to be present for these exams. Exceptions will be made only for extraordinary circumstances.

Grading: Your final grade will be based on your performance on the problem sets (20%), the two midterms (20% each), and the final exam (40%). We reserve the right to modify this scheme slightly.

Computers and calculators: The visualization of surfaces and other geometric phenomena is an important aspect of this course. Because computerized graphing programs will aid you in developing this ability, you are encouraged to employ them as part of the learning process. In this regard, we plan to have one or more resource people available to answer questions about Mathematica and possibly other software tools. (No prior experience with Mathematica is required.)

However, computers should be considered solely as an aid to the development of geometric intuition. In particular, the use of computers and other electronic aids will not be permitted during exams. (Bring only your brain and some pencils.) With this in mind, note that various homework problems ask you to sketch or otherwise describe various geometric objects. You are strongly advised to struggle with these first without electronic aids, as they may be quite trivial with a graphing program.

Words of Caution and Advice: This course may be more demanding than your previous mathematics courses at Harvard and elsewhere. In particular, the weekly assignments will be time-consuming and you should plan now to set aside regular hours to wrestle with them. It is also highly improbable that you will do well in this course without working through the daily assignments in order to stay current with the topics of the course. The pace of the course is somewhat fast, and new material builds on old. Thus, do not fall behind. If you find yourself falling behind, please contact your section teacher immediately to discuss options for help. Indeed, Harvard provides many services along these lines for its students, and your section teacher can help you to find them.

When you are working your assignments, keep in mind that your success in this course will require more than just memorizing formulas and "plugging in values". Numerical calculations are still important, but play a smaller role than in 1-variable calculus. Here is the key to success: Understand the underlying concepts and then work enough problems so that you can employ them in any example thrown at you. (In this regard, you will consistently battle with homework and exam problems that differ significantly from material discussed in class.)

Detailed syllabus (the dates shown may change somewhat as the course proceeds)

Week	Topics
Feb 5 - 7	Introduction to \mathbf{R}^2 and \mathbf{R}^3 . Points vs. vectors. Examples of curves and surfaces defined by algebraic equations. Vector-valued functions - parametrized curves in \mathbf{R}^2 and \mathbf{R}^3 . Equation(s) of a line. Velocity and acceleration vectors. Arclength. Dot product in \mathbf{R}^2 , \mathbf{R}^3 , and \mathbf{R}^n . Scalar and vector projections. Equation of a plane in \mathbf{R}^3 .
Feb 12 - 16	The cross product in \mathbf{R}^3 . Intersection of lines and planes. Introduction to row reduction techniques. Unit tangent T and unit normal vector N for a parametrized curve. Tangential and normal components of acceleration. Curvature. Equations of motion. Introduction to functions of several variables, especially functions of two and three variables. Graph of a function of two variables. Level curves (contours) of a function of two variables. Level surfaces of a function of three variables.
Feb 20 - 23	Vector fields in \mathbf{R}^2 and \mathbf{R}^3 . Examples from physics and differential equations. Integration of a function along a parametrized curve. Line integrals and work done by a variable force along a parametrized curve.
Feb 26 - Mar 2	Partial derivatives. Continuity and differentiability of functions of several variables. Linear approximation and tangent planes. Rate of change of a function along a parametrized curve. Basic chain rule. Directional derivative and the gradient vector. The gradient vector field of a function of several variables. Conservative vector fields, independence of path, and the Fundamental Theorem of Line Integrals.
Mar 5 - 9	Higher order derivatives & quadratic approx. Review for Midterm Exam I. Application: Partial differential equations in physics.
Mar 12 - 16	Extrema of functions of several variables. Unconstrained optimization. Constrained optimization and the method of Lagrange multipliers. Applications in economics.
Mar 19 - 23	General chain rule. Implicit differentiation. Economics applications. Integration over regions in \mathbf{R}^2 and \mathbf{R}^3 . Average value of a function. Iterated integrals and the Fubini Theorem.
Apr 2 - 6	Integrals in polar coordinates (\mathbf{R}^2) and cylindrical coordinates (\mathbf{R}^3). Integrals in cylindrical and spherical coordinates. Applications: centroids, center of mass, moment of inertia.
Apr 9 - 13	Change of variables in multiple integrals. Jacobian matrices. Review for Midterm Exam II. Line integrals and Green's theorem.
Apr 16 - 20	Parametrized surfaces in \mathbf{R}^3 . Surface integrals, surface area. Flux of a vector field through a surface.
Apr 23 - 27	Divergence of a vector field and the Divergence Theorem. Curl of a vector field and Stokes' Theorem. Review of Five Fundamental Theorems of Calculus.
Apr 30 - May 4	Applications to differential equations: The Heat Equation. Applications in physics: Maxwell's Equations. Last details.

The topics in the BioChem section will differ for approximately the last three weeks of the course.