DETAILED COURSE OUTLINE FOR MATH 21a

What follows is a rough, section by section outline of the material that we covered this semester in Math 21a. The outline for the sections in Chapter 5 of Ostebee and Zorn is only relevant for the Regular and Physics sections, while the outline for Chapters 2-5 of the Rosner text is only relevant for the BioChem sections.

Chapter 1.1:

- R^3 = 3-d space, R^2 = 2-d space.
- Abstract spaces versus coordinates.
- Drawing and labeling axis. Rotating axis.
- Distances and midpoints.
- Graphing equations.
- Simple surfaces, spheres, ellipses, parabolas, saddles.
- Orientations.
- Graphing in 2-d. Graphs as curves.
- Curves which aren't graphs.

Chapter 1.2 and Appendix A:

- Introduce curves in the plane as graphs of functions and pieces of graphs.
- Introduce parametrized curves in the plane.
- Parametrized curves in R^3. $t \to (x(t), y(t), z(t))$ for t ranging over some part of R.
- Explain the difference between a curve and its parametrization. The curve sits as a path in R^3; the parametrization tells the speed of an ant walking the path.
- Parametrization and reversing direction on the path.
- Polar coordinates and parametrized curves in polar coordinates.

Chapter 1.3:

- Vectors in the plane as arrows, or algebraically as 2-tuples (a, b). Vectors in space as arrows or 3-tuples (a, b, c).
- If a vector **v** = (a, b), then a and b are called the <u>components</u> of **v**. Likewise for a 3-d vector, **v** = (a, b, c).
- A vector, **v**, has a <u>length</u> or <u>magnitude</u>, $|\mathbf{v}| = (a^2 + b^2 + c^2)^{1/2}$.

- Adding two vectors, **v** + **w**. Multiplying by numbers (scalars) r **v** where r = 2, -.556, etc. Give graphical picture of addition and multiplication. (Also, multiplying by –1 to get –**v**.)
- The zero vector **0** = (0, 0) or (0, 0, 0) in 3-d.
- The triangle inequality |**v** + **w**| ≤ |**v**| + |**w**|.
- The <u>standard</u> <u>basis</u>, **i** = (1, 0, 0), **j** = (0, 1, 0), **k** = (0, 0, 1). Thus, any **v** = (a, b, c) can be <u>decomposed</u> as **v** = a **i** + b **j** + c **k** in terms of the standard basis. (We shall play with other basis as well later on.)

Chapter 1.4:

- Vector valued functions of time, t → **v**(t) = (a(t), b(t), c(t)). Thus, as t changes, the end point of **v**(t) traces out a parameterized curve in R^3.
- For example, the line segment running between (x_0, y_0) and (x_0 + a, y_0 + b) can be parametrized as (x_0, y_0) + t (a, b) as t runs between 0 and 1 (written t ∈ [0, 1]). Writing $\mathbf{P}_0$ = (x_0, y_0) and **v** = (a, b), this becomes **P**(t) = $\mathbf{P}_0$ + t **v**.
- The derivative of a vector valued function of time **v**(t) gives a new vector valued function of time, $\frac{d}{dt}\mathbf{v}(t) \equiv \mathbf{v}'(t) = (a'(t), b'(t), c'(t))$.
- The derivative **v**´(t) lives on a "different" version of R^3 as does **v**(t) since its tail sits naturally at **v**(t), not at the origin. This is because $\mathbf{v}'(t) = \lim_{\Delta t \to 0} (\mathbf{v}(t + \Delta t) - \mathbf{v}(t))/\Delta t$ and so is a vector which might naturally be interpreted as having its end, not at the origin, but at **v**(t).
- Lets use different notation: If we view t → **r**(t) as a vector valued function which traces out a path in space, then $\frac{d}{dt}\mathbf{r} \equiv \mathbf{r}'(t)$ is the instantaneous velocity at time t of the particle on the path; |**r**´(t)| is its instantaneous speed, and $\frac{d}{dt}\mathbf{r}'(t) = \mathbf{r}''(t)$ is its instantaneous acceleration of the particle.
- Algebraic rules: (**v**(t) + **w**(t))´ = **v**´(t) + **w**´(t) and (r(t) **v**(t))´ = r´(t) **v**(t) + r(t) **v**(t) when r is a function of t as well as **v**.
- Anti-derivatives of **v**(t). This is the vector $\mathbf{f}(t) = \int_0^t \mathbf{v}(\tau)\, d\tau$. If **v**(t) = (a(t), b(t), c(t)), then the vector **f** has components ($\int_0^t a(\tau)\, d\tau$, $\int_0^t b(\tau)\, d\tau$, $\int_0^t c(\tau)\, d\tau$).
- Thus, if **v**(t) = **r**´(t), then $\int_0^t \mathbf{v}(\tau)\, d\tau = \mathbf{r}(t) - \mathbf{r}(0)$.

Chapter 1.5:

- Newton's law: m **r**´´(t) = **F**.