# 9 Review

## • CONCEPT CHECK •

1. A scalar is a real number, while a vector is a quantity that has both a real-valued magnitude and a direction.
2. To add two vectors geometrically, we can use either the Triangle Law or the Parallelogram Law, as illustrated in Figures 3 and 4 in Section 9.2. (See also the definition of vector addition on page 653.) Algebraically, we add the corresponding components of the vectors.
3. For $c > 0$, $c\mathbf{a}$ is a vector with the same direction as $\mathbf{a}$ and length $c$ times the length of $\mathbf{a}$. If $c < 0$, $c\mathbf{a}$ points in the opposite direction as $\mathbf{a}$ and has length $|c|$ times the length of $\mathbf{a}$. (See Figure 7 in Section 9.2.) Algebraically, to find $c\mathbf{a}$ we multiply each component of $\mathbf{a}$ by $c$.
4. See (1) in Section 9.2.
5. See the definition on page 661 and the boxed equation on page 663.
6. The dot product can be used to determine the work done moving an object given the force and displacement vectors. The dot product can also be used to find the angle between two vectors and the scalar projection of one vector onto another. In particular, the dot product can determine if two vectors are orthogonal.
7. See the boxed equations on page 665 as well as Figures 5 and 6 and the accompanying discussion on pages 664-665.
8. See the definition on page 668; use either (2) or (4) in Section 9.4 .
9. The cross product can be used to determine torque if the force and position vectors are known. In addition, the cross product can be used to create a vector orthogonal to two given vectors as well as to determine if two vectors are parallel. The cross product can also be used to find the area of a parallelogram determined by two vectors.
10. (a) The area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ is the length of the cross product: $|\mathbf{a} \times \mathbf{b}|$.

    (b) The volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product: $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.
11. If an equation of the plane is known, it can be written as $ax + by + cz + d = 0$. A normal vector, which is perpendicular to the plane, is $\langle a, b, c\rangle$ (or any scalar multiple of $\langle a, b, c\rangle$). If an equation is not known, we can use points on the plane to find two non-parallel vectors which lie in the plane. The cross product of these vectors is a vector perpendicular to the plane.
12. The angle between two intersecting planes is defined as the acute angle between their normal vectors. We can find this angle using the definition of the dot product on page 661.
13. See (1), (2), and (3) in Section 9.5.
14. See (4), (5), and (6) in Section 9.5.
15. (a) Two (nonzero) vectors are parallel if and only if one is a scalar multiple of the other. In addition, two nonzero vectors are parallel if and only if their cross product is $\mathbf{0}$.

    (b) Two vectors are perpendicular if and only if their dot product is 0.

    (c) Two planes are parallel if and only if their normal vectors are parallel.
16. (a) Determine the vectors $\overrightarrow{PQ} = \langle a_1, a_2, a_3\rangle$ and $\overrightarrow{PR} = \langle b_1, b_2, b_3\rangle$. If there is a scalar $t$ such that $\langle a_1, a_2, a_3\rangle = t\langle b_1, b_2, b_3\rangle$, then the vectors are parallel and the points must all lie on the same line.
    Alternatively, if $\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{0}$, then $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel, so $P$, $Q$, and $R$ are collinear.
    Thirdly, an algebraic method is to determine an equation of the line joining two of the points, and then check whether or not the third point satisfies this equation.

(b) Find the vectors $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{PR} = \mathbf{b}$, $\overrightarrow{PS} = \mathbf{c}$. $\mathbf{a} \times \mathbf{b}$ is normal to the plane formed by $P$, $Q$ and $R$, and so $S$ lies on this plane if $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$ are orthogonal, that is, if $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0$. (Or use the reasoning in Example 6 in Section 9.4.)

Alternatively, find an equation for the plane determined by three of the points and check whether or not the fourth point satisfies this equation.

**17.** (a) See Exercise 9.4.27.

(b) See Example 8 in Section 9.5.

(c) See Example 10 in Section 9.5.

**18.** One method of graphing a function of two variables is to first find traces (see Example 6 in Section 9.6 and the discussion preceding it).

**19.** See Table 2 in Section 9.6.

**20.** (a) See (1) and the discussion accompanying Figure 3 in Section 9.7.

(b) See (3) and Figures 6–8, and the accompanying discussion, in Section 9.7.

## ▲ TRUE–FALSE QUIZ ▲

**1.** True, by Property 2 of the dot product. (See page 664).

**2.** False. Property 1 of the cross product says that $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$. (See page 669).

**3.** True. If $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$, then by definition of the cross product,
$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta = |\mathbf{v}|\,|\mathbf{u}| \sin\theta = |\mathbf{v} \times \mathbf{u}|$.
(Or, by Properties 1 and 2 of the cross product, $|\mathbf{u} \times \mathbf{v}| = |-\mathbf{v} \times \mathbf{u}| = |-1|\,|\mathbf{v} \times \mathbf{u}| = |\mathbf{v} \times \mathbf{u}|$.)

**4.** This is true by Property 4 of the dot product.

**5.** Property 2 of the cross product tells us that this is true.

**6.** This is true by Property 4 of the cross product.

**7.** This is true by (6) in Section 9.4.

**8.** In general, this assertion is false; a counterexample is $\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) \neq (\mathbf{i} \times \mathbf{i}) \times \mathbf{j}$. (See the discussion following Example 2 on page 669.

**9.** This is true because $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{u}$ (see page 668), and the dot product of two orthogonal vectors is 0.

**10.**
$$\begin{aligned}(\mathbf{u} + \mathbf{v}) \times \mathbf{v} &= \mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{v} \quad \text{(by Property 4 of the cross product)} \\ &= \mathbf{u} \times \mathbf{v} + \mathbf{0} \quad \text{(by the margin note on page 668)} \\ &= \mathbf{u} \times \mathbf{v}\text{, so this is true.}\end{aligned}$$

**11.** If $|\mathbf{u}| = 1$, $|\mathbf{v}| = 1$ and $\theta$ is the angle between these two vectors (so $0 \le \theta \le \pi$), then by definition of the cross product, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta = \sin\theta$, which is equal to 1 if and only if $\theta = \frac{\pi}{2}$ (that is, if and only if the two vectors are orthogonal). Therefore, the assertion that the cross product of two unit vectors is a unit vector is false.

**12.** This is false, because according to (7) in Section 9.5, $ax + by + cz + d = 0$ is the general equation of a plane.

**13.** This is false. In $\mathbb{R}^2$, $x^2 + y^2 = 1$ represents a circle, but $\{(x, y, z) \mid x^2 + y^2 = 1\}$ represents a *three-dimensional surface*, namely, a circular cylinder with axis the $z$-axis.

**14.** This is false, as the dot product of two vectors is a scalar, not a vector.

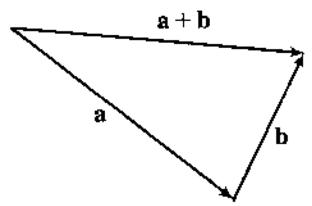
## EXERCISES

**1.** (a) The radius of the sphere is the distance between the points $(-1, 2, 1)$ and $(6, -2, 3)$, namely $\sqrt{[6-(-1)]^2 + (-2-2)^2 + (3-1)^2} = \sqrt{69}$. By the formula for an equation of a sphere (see page 650), an equation of the sphere with center $(-1, 2, 1)$ and radius $\sqrt{69}$ is $(x+1)^2 + (y-2)^2 + (z-1)^2 = 69$.

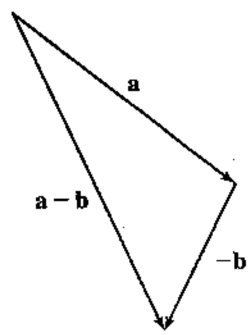
(b) The intersection of this sphere with the $yz$-plane is the set of points on the sphere whose $x$-coordinate is 0. Putting $x = 0$ into the equation, we have $(y-2)^2 + (z-1)^2 = 68, x = 0$ which represents a circle in the $yz$-plane with center $(0, 2, 1)$ and radius $\sqrt{68}$.

(c) Completing squares gives $(x-4)^2 + (y+1)^2 + (z+3)^2 = -1 + 16 + 1 + 9 = 25$. Thus, the sphere is centered at $(4, -1, -3)$ and has radius 5.

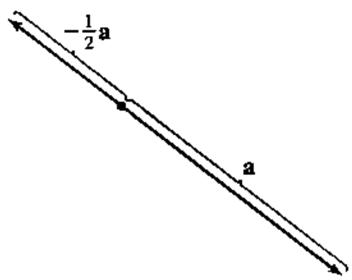
**2.** (a)



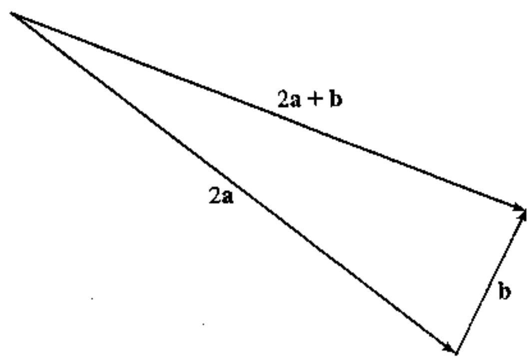

(b)




(c)




(d)




**3.** $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos 45° = (2)(3)\frac{\sqrt{2}}{2} = 3\sqrt{2}$. $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin 45° = (2)(3)\frac{\sqrt{2}}{2} = 3\sqrt{2}$. By the right-hand rule, $\mathbf{u} \times \mathbf{v}$ is directed out of the page.

**4.** (a) $2\mathbf{a} + 3\mathbf{b} = 2\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k} + 9\,\mathbf{i} - 6\,\mathbf{j} + 3\,\mathbf{k} = 11\,\mathbf{i} - 4\,\mathbf{j} - \mathbf{k}$

(b) $|\mathbf{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

(c) $\mathbf{a} \cdot \mathbf{b} = (1)(3) + (1)(-2) + (-2)(1) = -1$

(d) $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix} = (1-4)\,\mathbf{i} - (1+6)\,\mathbf{j} + (-2-3)\,\mathbf{k} = -3\,\mathbf{i} - 7\,\mathbf{j} - 5\,\mathbf{k}$

(e) $\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 0 & 1 & -5 \end{vmatrix} = 9\,\mathbf{i} + 15\,\mathbf{j} + 3\,\mathbf{k}$, $|\mathbf{b} \times \mathbf{c}| = 3\sqrt{9 + 25 + 1} = 3\sqrt{35}$

(f) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & 1 \\ 0 & 1 & -5 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & -5 \end{vmatrix} - \begin{vmatrix} 3 & 1 \\ 0 & -5 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} = 9 + 15 - 6 = 18$

(g) $\mathbf{c}\times\mathbf{c} = \mathbf{0}$ for any $\mathbf{c}$.

(h) From part (e),

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{a}\times(9\,\mathbf{i} + 15\,\mathbf{j} + 3\,\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ 9 & 15 & 3 \end{vmatrix} = (3+30)\,\mathbf{i} - (3+18)\,\mathbf{j} + (15-9)\,\mathbf{k}$$

$$= 33\,\mathbf{i} - 21\,\mathbf{j} + 6\,\mathbf{k}.$$

(i) The scalar projection is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = |\mathbf{b}|\cos\theta = \mathbf{a}\cdot\mathbf{b}/|\mathbf{a}| = -\frac{1}{\sqrt{6}}$.

(j) The vector projection is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = -\frac{1}{\sqrt{6}}(\mathbf{a}/|\mathbf{a}|) = -\frac{1}{6}(\mathbf{i} + \mathbf{j} - 2\,\mathbf{k})$.

(k) $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{-1}{\sqrt{6}\sqrt{14}} = \dfrac{-1}{2\sqrt{21}}$ and $\theta = \cos^{-1}\dfrac{-1}{2\sqrt{21}} \approx 96^\circ$.

**5.** For the two vectors to be orthogonal, we need $\langle 3, 2, x\rangle \cdot \langle 2x, 4, x\rangle = 0 \quad\Leftrightarrow$ $(3)(2x) + (2)(4) + (x)(x) = 0 \quad\Leftrightarrow\quad x^2 + 6x + 8 = 0 \quad\Leftrightarrow\quad (x+2)(x+4) = 0 \quad\Leftrightarrow\quad x = -2$ or $x = -4$.

**6.** We know that the cross product of two vectors is orthogonal to both. So we calculate $(\mathbf{j} + 2\,\mathbf{k})\times(\mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k}) = [3 - (-4)]\,\mathbf{i} - (0 - 2)\,\mathbf{j} + (0 - 1)\,\mathbf{k} = 7\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$. Then two unit vectors orthogonal to both given vectors are $\pm\dfrac{7\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}}{\sqrt{7^2 + 2^2 + (-1)^2}} = \pm\dfrac{1}{3\sqrt{6}}(7\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$, that is, $\frac{7}{3\sqrt{6}}\,\mathbf{i} + \frac{2}{3\sqrt{6}}\,\mathbf{j} - \frac{1}{3\sqrt{6}}\,\mathbf{k}$ and $-\frac{7}{3\sqrt{6}}\,\mathbf{i} - \frac{2}{3\sqrt{6}}\,\mathbf{j} + \frac{1}{3\sqrt{6}}\,\mathbf{k}$.

**7.** (a) $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = 2$

(b) $\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v}) = \mathbf{u}\cdot[-(\mathbf{v}\times\mathbf{w})] = -\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = -2$

(c) $\mathbf{v}\cdot(\mathbf{u}\times\mathbf{w}) = (\mathbf{v}\times\mathbf{u})\cdot\mathbf{w} = -(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = -2$

(d) $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v} = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{v}) = \mathbf{u}\cdot\mathbf{0} = 0$

**8.**
$$\begin{aligned}(\mathbf{a}\times\mathbf{b})\cdot[(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a})] &= (\mathbf{a}\times\mathbf{b})\cdot\big([(\mathbf{b}\times\mathbf{c})\cdot\mathbf{a}]\,\mathbf{c} - [(\mathbf{b}\times\mathbf{c})\cdot\mathbf{c}]\,\mathbf{a}\big) \\ &\qquad \text{(see Exercise 9.4.30)} \\ &= (\mathbf{a}\times\mathbf{b})\cdot[(\mathbf{b}\times\mathbf{c})\cdot\mathbf{a}]\,\mathbf{c} = [\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]\,(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} \\ &= [\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]\,[\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})] = [\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})]^2\end{aligned}$$

**9.** For simplicity, consider a unit cube positioned with its back left corner at the origin. Vector representations of the diagonals joining the points $(0,0,0)$ to $(1,1,1)$ and $(1,0,0)$ to $(0,1,1)$ are $\langle 1,1,1\rangle$ and $\langle -1,1,1\rangle$. Let $\theta$ be the angle between these two vectors. $\langle 1,1,1\rangle\cdot\langle -1,1,1\rangle = -1 + 1 + 1 = 1 = |\langle 1,1,1\rangle|\,|\langle -1,1,1\rangle|\cos\theta = 3\cos\theta$ $\Rightarrow\quad \cos\theta = \frac{1}{3} \quad\Rightarrow\quad \theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 71^\circ$.

**10.** $\overrightarrow{AB} = \langle 1,3,-1\rangle$, $\overrightarrow{AC} = \langle -2,1,3\rangle$ and $\overrightarrow{AD} = \langle -1,3,1\rangle$. By Equation 9.4.7,

$\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right) = \begin{vmatrix} 1 & 3 & -1 \\ -2 & 1 & 3 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} - 3\begin{vmatrix} -2 & 3 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} = -8-3+5 = -6$. The volume is

$\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right| = 6$ cubic units.

**11.** $\overrightarrow{AB} = \langle 1,0,-1\rangle$, $\overrightarrow{AC} = \langle 0,4,3\rangle$, so

(a) a vector perpendicular to the plane is $\overrightarrow{AB}\times\overrightarrow{AC} = \langle 0+4, -(3+0), 4-0\rangle = \langle 4,-3,4\rangle$.

(b) $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\sqrt{16+9+16} = \frac{\sqrt{41}}{2}$.

**12.** $\mathbf{D} = 4\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k}$, $W = \mathbf{F}\cdot\mathbf{D} = 12 + 15 + 60 = 87$ joules

**13.** Let $F_1$ be the magnitude of the force directed $20°$ away from the direction of shore, and let $F_2$ be the magnitude of the other force. Separating these forces into components parallel to the direction of the resultant force and perpendicular to it gives $F_1\cos 20° + F_2\cos 30° = 255$ (1), and $F_1 \sin 20° - F_2\sin 30° = 0 \Rightarrow$ $F_1 = F_2\dfrac{\sin 30°}{\sin 20°}$ (2). Substituting (2) into (1) gives $F_2(\sin 30°\cot 20° + \cos 30°) = 255 \Rightarrow F_2 \approx 114$ N. Substituting this into (2) gives $F_1 \approx 166$ N.

**14.** $|\boldsymbol{\tau}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = (0.40)(50)\sin(90° - 30°) \approx 17.3$ joules

**15.** $x = 1 + 2t$, $y = 2 - t$, $z = 4 + 3t$

**16.** $\mathbf{v} = \langle 8,-2,5\rangle$, so $x = -6 + 8t$, $y = -1 - 2t$ and $z = 5t$.

**17.** $\mathbf{v} = \langle 4,-3,5\rangle$, so $x = 1 + 4t$, $y = -3t$, $z = 1 + 5t$.

**18.** $2(x-4) + 6(y+1) - 3(z+1) = 0$ or $2x + 6y - 3z = 5$.

**19.** Since the two planes are parallel, they will have the same normal vectors. So se can take $\mathbf{n} = \langle 1,2,5\rangle$ and an equation of the plane is $1\,[x-(-4)] + 2(y-1) + 5(z-2) = 0$ or $x + 2y + 5z = 8$.

**20.** Here the vectors $\mathbf{a} = \langle 2-(-1), 0-2, 1-0\rangle = \langle 3,-2,1\rangle$ and $\mathbf{b} = \langle -5-(-1), 3-2, 1-0\rangle = \langle -4,1,1\rangle$ lie in the plane, so $\mathbf{n} = \mathbf{a}\times\mathbf{b} = \langle -3,-7,-5\rangle$ is a normal vector to the plane and an equation of the plane is $-3\,[x-(-1)] - 7\,(y-2) - 5\,(z-0) = 0$ or $3x + 7y + 5z = 11$.

**21.** $\mathbf{n}_1 = \langle 1,0,-1\rangle$ and $\mathbf{n}_2 = \langle 0,1,2\rangle$. Setting $z = 0$, it is easy to see that $(1,3,0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v}_1 = \mathbf{n}_1\times\mathbf{n}_2 = \langle 1,-2,1\rangle$. A second vector parallel to the desired plane is $\mathbf{v}_2 = \langle 1,1,-2\rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, the normal of the plane in question is $\mathbf{n} = \mathbf{v}_1\times\mathbf{v}_2 = \langle 4-1, 1+2, 1+2\rangle = 3\,\langle 1,1,1\rangle$. Taking $(x_0, y_0, z_0) = (1,3,0)$, the equation we are looking for is $(x-1) + (y-3) + z = 0 \Leftrightarrow x + y + z = 4$.

**22.** Substitution of the parametric equations into the equation of the plane gives $2x - y + z = 2(2-t) - (1+3t) + 4t = 2 \Rightarrow -t + 3 = 2 \Rightarrow t = 1$. When $t = 1$, the parametric equations give $x = 2 - 1 = 1$, $y = 1 + 3 = 4$ and $z = 4$. Therefore, the point of intersection is $(1,4,4)$.

**23.** Since the direction vectors $\langle 2,3,4\rangle$ and $\langle 6,-1,2\rangle$ aren't parallel, neither are the lines. For the lines to intersect, the three equations $1 + 2t = -1 + 6s$, $2 + 3t = 3 - s$, $3 + 4t = -5 + 2s$ must be satisfied simultaneously. Solving the first two equations gives $t = \frac{1}{5}$, $s = \frac{2}{5}$ and checking we see these values don't satisfy the third equation. Thus the lines aren't parallel and they don't intersect, so they must be skew.

**24.** (a) The normal vectors are $\langle 1,1,-1\rangle$ and $\langle 2,-3,4\rangle$. Since these vectors aren't parallel, neither are the planes parallel. Also $\langle 1,1,-1\rangle\cdot\langle 2,-3,4\rangle = 2 - 3 - 4 = -5 \neq 0$ so the normal vectors, and thus the planes, are not perpendicular.

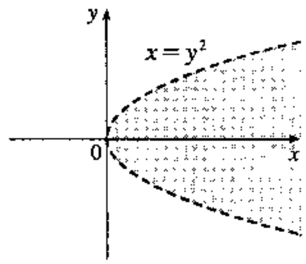
(b) $\cos\theta = \dfrac{\langle 1,1,-1\rangle \cdot \langle 2,-3,4\rangle}{\sqrt{3}\sqrt{29}} = -\dfrac{5}{\sqrt{87}}$ and $\theta = \cos^{-1}\left(-\frac{5}{\sqrt{87}}\right) \approx 122°$ (or we can say $\approx 58°$).

**25.** By Exercise 9.5.49, $D = \dfrac{|2-24|}{\sqrt{26}} = \dfrac{22}{\sqrt{26}}$.

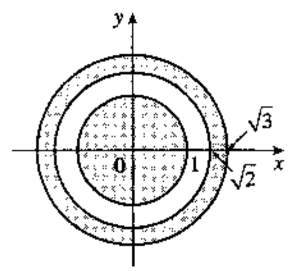
**26.** Use the formula proven in Exercise 9.4.27. In the notation used in that exercise, $\mathbf{a}$ is just the direction of the line; that is, $\mathbf{a} = \langle 1,-1,2\rangle$. A point on the line is $(1,2,-1)$ (setting $t=0$), and therefore $\mathbf{b} = \langle 1-0, 2-0, -1-0\rangle = \langle 1,2,-1\rangle$. Hence

$$d = \frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|} = \frac{|\langle 1,-1,2\rangle \times \langle 1,2,-1\rangle|}{\sqrt{1+1+4}} = \frac{|\langle -3,3,3\rangle|}{\sqrt{6}} = \sqrt{\frac{27}{6}} = \frac{3}{\sqrt{2}}.$$

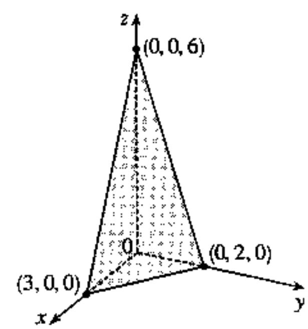
**27.** $\ln(x-y^2)$ is defined only when $x-y^2>0$, or $x>y^2$, and $x$ is defined for all real numbers, so the domain of the product $x\ln(x-y^2)$ is $\{(x,y) \mid x>y^2\}$.



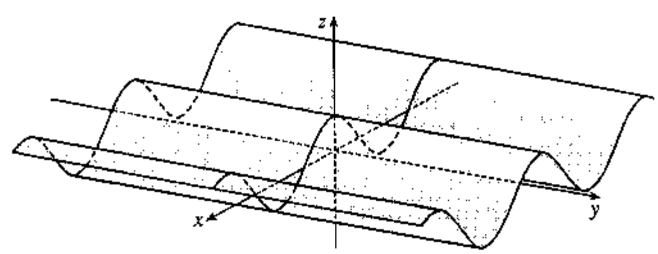

**28.** We need $\sin\pi(x^2+y^2) \geq 0 \;\Leftrightarrow\; 2n\pi \leq \pi(x^2+y^2) \leq (2n+1)\pi$, $n$ an integer, so $D = \{(x,y) \mid 2n \leq x^2+y^2 \leq 2n+1, n \text{ an integer}\}$.



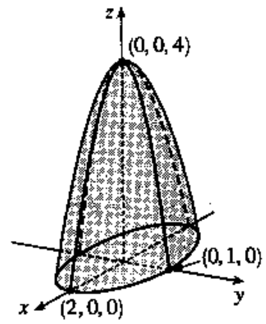

**29.** The graph is the plane $z = 6-2x-3y \;\Rightarrow\; 2x+3y+z=6$. The intercepts with the coordinate axes are $(3,0,0)$, $(0,2,0)$, and $(0,0,6)$ which enable us to sketch the portion of the plane that lies in the first octant.



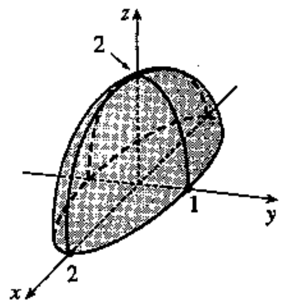

**30.** The equation is $z=\cos x$, which doesn't involve $y$. Thus the traces in $y=k$ are the graph $z=\cos x$, $y=k$, giving a cylindrical surface.



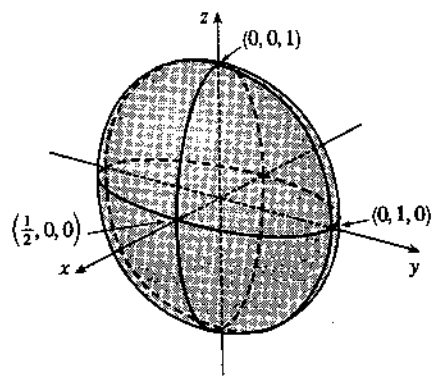
**31.** The equation is $z = 4 - x^2 - 4y^2$. The traces in $x = k$ are $z = 4 - k^2 - 4y^2$, a family of parabolas opening downward, as are the traces in $y = k$, $z = 4 - 4k^2 - x^2$. The traces in $z = k$ are $x^2 + 4y^2 = 4 - k$, a family of ellipses, so the surface is an elliptic paraboloid.



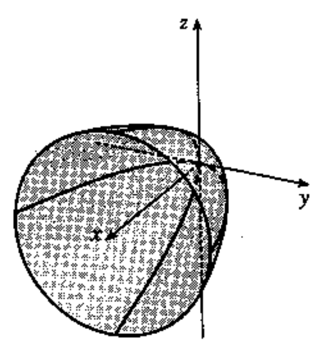

**32.** The equation is $z = \sqrt{4 - x^2 - 4y^2}$ or $x^2 + 4y^2 + z^2 = 4$, $z \geq 0$. The traces in $x = k$, $y = k$, and $z = k$ are ellipses or portions of ellipses, and the equation can be recognized as that of an ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{4} = 1$, $z \geq 0$, with intercepts $\pm 2$, $\pm 1$, and 2 for $x$, $y$, and $z$ respectively. Since $z \geq 0$, we have only the upper half of the ellipsoid.



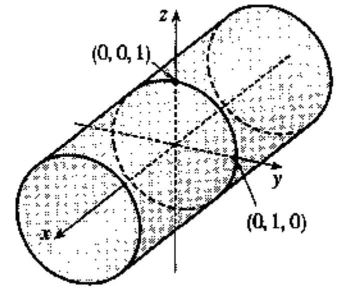

**33.** An equivalent equation is $\dfrac{x^2}{(1/2)^2} + y^2 + z^2 = 1$, an ellipsoid centered at the origin with intercepts $\pm\frac{1}{2}$, $\pm 1$, and $\pm 1$.



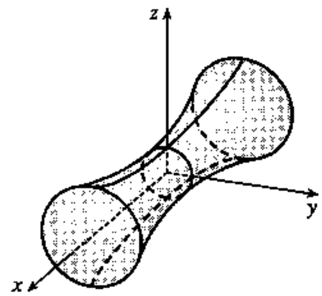

**34.** $x = y^2 + z^2$ is the equation of a circular paraboloid opening in the direction of the positive $x$-axis.



**35.** $y^2 + z^2 = 1$ is the equation of a circular cylinder with axis the $x$-axis.




**36.** An equivalent equation is $-x^2 + y^2 + z^2 = 1$, a hyperboloid of one sheet with axis the $x$-axis.




**37.** $x = 2\cos\frac{\pi}{6} = \sqrt{3}$, $y = 2\sin\frac{\pi}{6} = 1$, $z = 2$, so in rectangular coordinates the point is $\left(\sqrt{3}, 1, 2\right)$. $\rho = \sqrt{3+1+4} = 2\sqrt{2}$, $\theta = \frac{\pi}{6}$, and $\cos\phi = z/\rho = \frac{1}{\sqrt{2}}$, so $\phi = \frac{\pi}{4}$ and the spherical coordinates are $\left(2\sqrt{2}, \frac{\pi}{6}, \frac{\pi}{4}\right)$.

**38.** $r = \sqrt{4+4} = 2\sqrt{2}$, $z = -1$, $\cos\theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$ so $\theta = \frac{\pi}{4}$ and in cylindrical coordinates the point is $\left(2\sqrt{2}, \frac{\pi}{4}, -1\right)$. $\rho = \sqrt{4+4+1} = 3$, $\cos\phi = -\frac{1}{3}$, so the spherical coordinates are $\left(3, \frac{\pi}{4}, \cos^{-1}\left(-\frac{1}{3}\right)\right)$.

**39.** $x = 4\sin\frac{\pi}{6}\cos\frac{\pi}{3} = 1$, $y = 4\sin\frac{\pi}{6}\sin\frac{\pi}{3} = \sqrt{3}$, $z = 4\cos\frac{\pi}{6} = 2\sqrt{3}$ so in rectangular coordinates the point is $\left(1, \sqrt{3}, 2\sqrt{3}\right)$. $r^2 = x^2 + y^2 = 4$, $r = 2$, so the cylindrical coordinates are $\left(2, \frac{\pi}{3}, 2\sqrt{3}\right)$.

**40.** (a) $\theta = \frac{\pi}{4}$. In spherical coordinates, this is a half-plane including the $z$-axis and intersecting the $xy$-plane in the half-line $x = y$, $x > 0$.

(b) $\phi = \frac{\pi}{4}$. This is one frustum of a circular cone with vertex the origin and axis the positive $z$-axis.

**41.** $x^2 + y^2 + z^2 = 4$. In cylindrical coordinates, this becomes $r^2 + z^2 = 4$. In spherical coordinates, it becomes $\rho^2 = 4$ or $\rho = 2$.

**42.** $x^2 + y^2 = 4$. In cylindrical coordinates: $r^2 = 4$. In spherical coordinates: $\rho^2 - z^2 = 4$ or $\rho^2 - \rho^2\cos^2\phi = 4$ or $\rho^2\sin^2\phi = 4$ or $\rho\sin\phi = 2$.

**43.** The resulting surface is a circular paraboloid with equation $z = 4x^2 + 4y^2$. Changing to cylindrical coordinates we have $z = 4\left(x^2 + y^2\right) = 4r^2$.

**44.** $\rho = 2\cos\phi \;\Rightarrow\; \rho^2 = 2\rho\cos\phi \;\Rightarrow\; x^2 + y^2 + z^2 = 2z \;\Rightarrow\; x^2 + y^2 + (z-1)^2 = 1$. This is the equation of a sphere with radius 1, centered at $(0, 0, 1)$. Therefore, $0 \le \rho \le 2\cos\phi$ is the solid ball whose boundary is this sphere. $0 \le \theta \le \frac{\pi}{2}$ and $0 \le \phi \le \frac{\pi}{6}$ restrict the solid to the section of this ball that lies above the cone $\phi = \frac{\pi}{6}$ and is in the first octant.