**Last Name:** ______________________________

**First Name:** ______________________________

# Math 21a Exam #1: Tuesday, February 29, 2000

**SECTION (CIRCLE ONE):**

Robert Winters
John Friedman (CA)
MWF 10-11

Marty Weissman
Lukasz Fidkowski (CA)
MWF 11-12

Tom Weston
Nick Stang (CA)
MWF 11-12

Russell Mann
Mike Grobis (CA)
MWF 11-12 (Physics)

Tom Weston
James Griffin (CA)
MWF 12-1 (BioChem)

Yang Liu
Bartok Czech (CA)
TTh 10-11:30

Daniel Allcock
Nikhil Dutta (CA)
TTh 11:30-1

| Question | Points | Score |
|---|---|---|
| 1 | 16 | |
| 2 | 18 | |
| 3 | 16 | |
| 4 | 18 | |
| 5 | 14 | |
| 6 | 18 | |
| **Total** | 100 | |

The time allotted for this exam is 90 minutes.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

If more space is needed, use the back of the previous page and make note of this.

Please write neatly. Answers which are deemed illegible by the grader will not receive credit.

No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students.

*In agreeing to take this exam, you are implicitly accepting Harvard University's Honor Code.*

(1) a) Give parametric equations for the line of intersection of the planes
$2x + 2y - z = 15$ and $5x + 3y - 3z = 32$.

b) What angle does the line in a) make with the positive z-axis?

c) Find the point on the plane $2x + 2y - z = 15$ closest to the point (-1,1,3).

**(2)** Let M = (2, 0, 0), N = (3, 3, -1), P = ( -1, -2, 1), Q = (1, 3, 7) be points in $\mathbf{R}^3$.

a) Find parametric equations for the line containing M and Q.

b) Find an equation for the plane containing M, N, and P.

c) Find the area of the triangle MNP.

d) Find the volume of the pyramid MNPQ.
(You may use the fact that Volume = 1/3(area base)($\perp$ height) if you wish.)

**(3)** Let $\mathbf{p}(t) = (e^{-t}\cos t, e^{-t}\sin t, 10e^{-t})$ be the position of a particle at time $t$.

(a) Compute the velocity $\mathbf{v}$ as a function of $t$.

(b) Calculate the distance traveled by the particle as $t$ goes from 0 to $\infty$. [The distance will be finite.]

(4) Given the vectors **u** = (-1,2,3), **v** = (1,-2,1) , and **r** = (1,0,2), find the following quantities and complete the box. Show all work below or on the facing page.

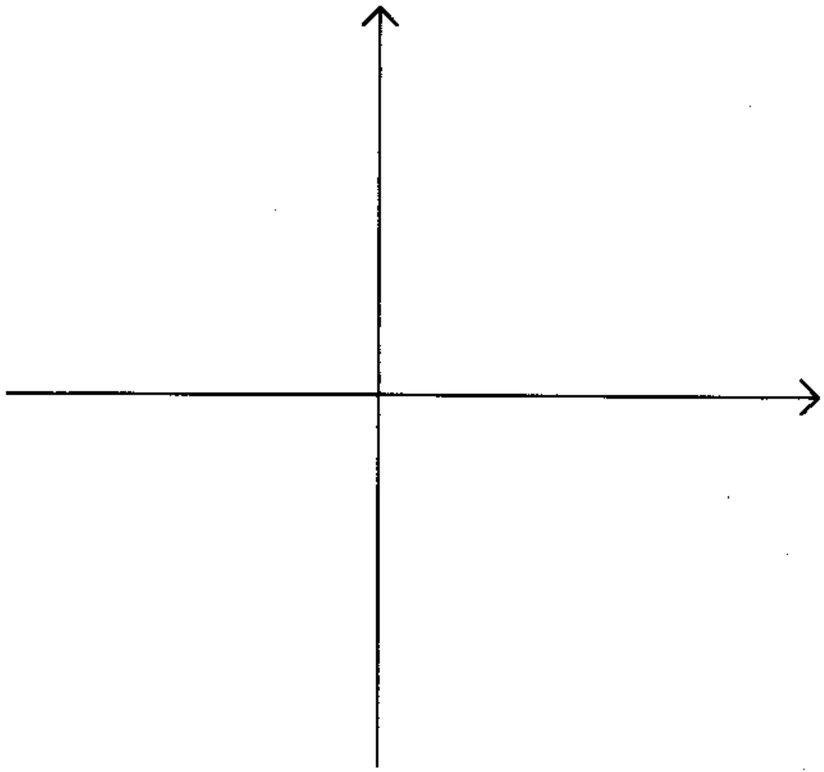
| | | |
|---|---|---|
| **u**·**v** = | The component (scalar projection) of **u** in the direction of **r** = | **u** × **v** = |
| 3**u** - 2**v** = | The vector projection of **u** in the direction of **r** = | The area of the parallelogram formed by **u** and 2**v** = |
| \|\|-2**r**\|\| = | The angle between the vectors **u** and **r** = | The volume of the parallelepiped with **u**, 2**v**, and 3**r** as three edges = |

**(5)** Show that if **a**, **b**, **c**, and **d** are positions of the vertices of a quadrilateral (four-sided figure), not necessarily lying in a plane, then the midpoints of the four sides are vertices of a parallelogram.

**(6)** a) Let L be a line in $\mathbf{R}^3$ passing through the origin and parallel to the vector $\mathbf{v} = \mathbf{j} + \mathbf{k}$, i.e. $\mathbf{v} = (0, 1, 1)$. Let P = $(x, y, z)$ be a point that is not on the line L. Find an expression in terms of $x$, $y$, and $z$ for the (shortest) distance from P to L.

b) Find an equation for the radius 1 cylinder with the line L as its central axis and with a radius of 1. (Hint: What property defines a cylinder geometrically?)

c) Draw an $x = 0$ cross section of the cylinder. Label and number your axes.



d) Draw a $y = 0$ cross section of the cylinder. Label and number your axes.