13 Review

• CONCEPT CHECK •

1. See Definitions 1 and 2 in Section 13.1. A vector field can represent, for example, the wind velocity at any location in space, the speed and direction of the ocean current at any location, or the force vectors of Earth's gravitational field at a location in space.

2. (a) A conservative vector field $\mathbf{F}$ is a vector field which is the gradient of some scalar function f.

(b) The function f in part (a) is called a potential function for $\mathbf{F}$, that is, $\mathbf{F} = \nabla f$.

3. (a) See Definition 13.2.2.

(b) We normally evaluate the line integral using Formula 13.2.3.

(c) The mass is $m = \int_C \rho(x, y)\, ds$, and the center of mass is $(\overline{x}, \overline{y})$ where $\overline{x} = \frac{1}{m} \int_C x\rho(x, y)\, ds$, $\overline{y} = \frac{1}{m} \int_C y\rho(x, y)\, ds$.

(d) See (5) and (6) in Section 13.2 for plane curves; we have similar definitions when C is a space curve (see the equation preceding (10) on page 930).

(e) For plane curves, see Equations 13.2.7. We have similar results for space curves (see the equation preceding (10) on page 930).

4. (a) See Definition 13.2.13.

(b) If $\mathbf{F}$ is a force field, $\int_C \mathbf{F} \cdot d\mathbf{r}$ represents the work done by $\mathbf{F}$ in moving a particle along the curve C.

(c) $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\, dx + Q\, dy + R\, dz$

5. See Theorem 13.3.2.

6. (a) $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path if the line integral has the same value for any two curves that have the same initial and terminal points.

(b) See Theorem 13.3.4.

7. See the statement of Green's Theorem on page 945.

8. See Equations 13.4.5.

9. (a) $\operatorname{curl} \mathbf{F} = \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x} \right) \mathbf{j} + \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \right) \mathbf{k} = \nabla \times \mathbf{F}$

(b) $\operatorname{div} \mathbf{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z} = \nabla \cdot \mathbf{F}$

(c) For curl $\mathbf{F}$, see the discussion accompanying Figure 1 on page 955 as well as Figure 6 and the accompanying discussion on page 975. For div $\mathbf{F}$, see the discussion following Example 5 on page 956 as well as the discussion preceding (8) on page 982.

10. See Theorem 13.3.6; see Theorem 13.5.4.

11. (a) See (1) in Section 13.6.

(b) We normally evaluate the surface integral using Formula 13.6.2.

(c) See Formula 13.6.4.

(d) The mass is $m = \iint_S \rho(x, y, z)\, dS$ and the center of mass is $(\overline{x}, \overline{y}, \overline{z})$ where $\overline{x} = \frac{1}{m} \iint_S x\rho(x, y, z)\, dS$, $\overline{y} = \frac{1}{m} \iint_S y\rho(x, y, z)\, dS$, $\overline{z} = \frac{1}{m} \iint_S z\rho(x, y, z)\, dS$.

12. (a) See Figures 6 and 7 and the accompanying discussion in Section 13.6. A Möbius strip is a nonorientable surface; see Figures 4 and 5 and the accompanying discussion on page 964.

(b) See Definition 13.6.8.

(c) See Formula 13.6.9.

(d) See Formula 13.6.10.

13. See the statement of Stokes' Theorem on page 971.

14. See the statement of the Divergence Theorem on page 978.

15. In each theorem, we have an integral of a "derivative" over a region on the left side, while the right side involves the values of the original function only on the boundary of the region.

▲ TRUE–FALSE QUIZ ▲

1. False; div **F** is a scalar field.

2. True. (See Definition 13.5.1.)

3. True, by Theorem 13.5.3 and the fact that div $\mathbf{0} = 0$.

4. True, by Theorem 13.3.2.

5. False. See Exercise 13.3.33. (But the assertion is true if D is simply-connected; see Theorem 13.3.6.)

6. False. See the discussion accompanying Figure 8 on page 929.

7. True. Apply the Divergence Theorem and use the fact that div $\mathbf{F} = 0$.

8. False by Theorem 13.5.11, because if it were true, then div curl $\mathbf{F} = 3 \neq 0$.