Last Name: ______________________________

First Name: ______________________________

Math 21a Exam #2: Wednesday, April 11, 2001

SECTION (CIRCLE ONE):

Robert Winters Michael Simonetti (CA) MWF 10-11	Robert Winters Lisa Carlivati (CA) MWF 11-12	Paul Bamberg Geoff Werner-Allen (CA) MWF 11-12 (Physics)
Dima Arinkin Nathan Moore (CA) MWF 12-1 (BioChem)	Mark Lucianovic Bryden Cais (CA) TTh 10-11:30	Yang Liu Anthony Varilly (CA) TTh 11:30-1

Question	Points	Score
1	16	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
Total	100	

The time allotted for this exam is 90 minutes.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

If more space is needed, use the back of the previous page and make note of this.

Please write neatly. Answers which are deemed illegible by the grader will not receive credit.

No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students.

(1) The President has opened up a wildlife preserve in Alaska for oil drilling. In retaliation, Democrats introduce a bill to allow oil drilling on the Ellipse, a park between the White House and the Washington Monument bounded by the curve $x^2 + 4y^2 = 100$. Each political party is to be given one drilling site.

The President consults geologists from Texas, who inform him that the value V of the oil from a well drilled on the Ellipse will be given by the formula $V = 200 + 18y - x^2 - y^2$.

(a) He thereupon signs the bill into law and instructs you, his Secretary of Energy, to find the coordinates x and y for the most valuable drilling site(s) (for the Republicans) and of the least valuable site(s) (for the Democrats) and to tell him the maximum and minimum values. Do so.

(b) The President also wants to know whether the Republicans could do even better if allowed to drill outside the Ellipse? Determine if there is a drilling site with even greater V, and if so, where it is located.

(2) The function $F(x,y) = x^2 y - 4xy + 3x^2 + \frac{1}{2}y^2$ has three stationary points, at $x = 0$, 1, and 5.

(a) Find the values of y at these three stationary points.

(b) Classify each stationary point as a maximum, minimum, or saddle point.

(3) A function $F(x, y)$ is given by the formula $F(x, y) = g(x^2 + y^2)$, where g is a twice-differentiable function of one variable. Express $\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}$ in terms of x, y, and the first and second derivatives of g.

(4) Cartesian and polar coordinates are related by $\begin{Bmatrix} x = r\cos\theta \\ y = r\sin\theta \end{Bmatrix}$. Suppose that at a certain instant in time, a particle is located at $(x, y) = (3, 4)$, and its polar coordinates are changing as specified by $\frac{dr}{dt} = 2$ and $\frac{d\theta}{dt} = 1$. Use the chain rule to calculate $\frac{dx}{dt}$ and $\frac{dy}{dt}$ for the particle at this instant.

(5) (a) Using Cartesian coordinates, evaluate the integral of the function $x^2 + y^2$ over the right triangle with vertices $(x, y) = (0, 0)$, $(a, 0)$, and (a, a).

(b) Evaluate the same integral using polar coordinates. (Hint: make the substitution $u = \tan\theta$.)

(6) Convert the integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} e^{-(x^2+y^2)}dydx$ to polar coordinates and hence evaluate it exactly. Sketch the region R over which the integration is being performed.

(7) Suppose that a mass density function is given by $\delta(x, y, z) = x + z$. Set up, but do not evaluate, an iterated integral for the mass of the body which has this density function and which is bounded by the surfaces $x^2 + y^2 = 4$, $x + y + z = 5$, and $z = 1$.