

Biöchem solutions  
(Questions 6-10 on Biöchem test)

## Math 21a Final Exam

spring 2002

Question 3 (8 points total)

### Grading

- (a) (5 points)  
direction of  $\nabla f(4, 5, 6)$ : 1 point  
length of  $\nabla f(4, 5, 6)$ : 1 point  
direction of  $u$ : 1 point  
length of  $u$ : 1 point  
taking directional derivative: 1 point
- (b) (3 points)

Question 6 (8 points total) The scores of Math21a final exam of a section of 9 students are as follows:

$i$	1	2	3	4	5	6	7	8	9
$x_i$	45	59	82	67	58	63	22	56	52

The arithmetic mean is 56, and the sample variance is 269.

- (a) (2 points) What is the sample median?
- (b) (2 points) The arithmetic mean is so low that the course head decided to rescale and translate the scores. The adjusted score  $y_i$  and the original score  $x_i$  are related by  $y_i = cx_i + d$ , where  $c, d$  are chosen such that the highest adjusted score is 90 and the lowest adjusted score is 60. Find  $c, d$ .
- (c) (4 points) Find the arithmetic mean and sample variance of the adjusted scores.

### Solution

- (a)  $22 \leq 45 \leq 52 \leq 56 \leq 58 \leq 59 \leq 63 \leq 67 \leq 82$   
sample median = 58.
- (b)  $82c + d = 90, 22 + d = 60 \Rightarrow c = 0.5, d = 49$ .

- (c)  $y_i = 0.5x_i + 49$   
 arithmetic mean =  $0.5 \times 56 + 49 = 77$   
 sample variance =  $(0.5)^2 \times 269 = 67.25$

**Question 7** (12 points total) A study was conducted of risk factors for HIV infection among intravenous drug users. It was found that 40% of the light users and 55% of the heavy users are HIV positive.

Use TABLE 1 at the end of this exam to answer (a), (b), (c).

- (a) (2 points) What is the probability that exactly 2 of 6 light users are HIV positive?  
 (b) (2 points) What is the probability that at least 2 of 6 light users are HIV positive?  
 (c) (2 points) What is the probability that exactly 4 of 7 heavy users are HIV positive?

Suppose that among the intravenous drug users, 70% are light users, and 30% are heavy users.

- (d) (3 points) What is the probability that a random intravenous drug user is HIV positive?  
 (e) (3 points) What is the probability that an HIV negative intravenous drug user is a light user? (Express your answer as a fraction in lowest terms)

**Solution** Let  $X$  be the number of HIV positive users in 6 light users. Then  $X \sim \text{BIN}(6, 0.4)$ .

- (a)  $Pr(X = 2) = 0.3110$   
 (b)

$$\begin{aligned} Pr(X \geq 2) &= 1 - Pr(X = 0) - Pr(X = 1) \\ &= 1 - 0.0467 - 0.1866 = 0.7667 \end{aligned}$$

- (c) Let  $Y$  be the number of HIV negative users in 7 heavy users. Then  $Y \sim \text{BIN}(7, 0.45)$ .  $Pr(Y = 3) = 0.2918$   
 (d) Let  $A = \text{HIV positive}$ ,  $\bar{A} = \text{HIV negative}$ ,  $B = \text{light user}$ ,  $\bar{B} = \text{heavy user}$ . Then  $Pr(A|B) = 0.4$ ,  $Pr(A|\bar{B}) = 0.55$ ,  $Pr(B) = 0.7$ , and  $Pr(\bar{B}) = 0.3$ .

$$Pr(A) = Pr(A|B)Pr(B) + Pr(A|\bar{B})Pr(\bar{B}) = 0.4 \times 0.7 + 0.55 \times 0.3 = 0.445$$

(e)

$$\begin{aligned}Pr(B|\bar{A}) &= \frac{Pr(\bar{A}|B)Pr(B)}{Pr(\bar{A}|B)Pr(B) + Pr(\bar{A}|\bar{B})Pr(\bar{B})} \\ &= \frac{0.6 \times 0.7}{0.6 \times 0.7 + 0.45 \times 0.3} \\ &= \frac{28}{37}\end{aligned}$$

Question 8 (9 points total) Assume that in 45-54 year-old men, FEV (forced expiratory volume) is normally distributed with mean 4.0 liters and standard deviation 0.5 liter. Use TABLE 3 at the end of the exam to answer the following questions (a), (b), (c).

- (3 points) What is the probability that a 45-54 year-old man has an FEV greater than 4.8?
- (3 points) What is the probability that a 45-54 year-old man has an FEV less than 3.5?
- (3 points) Find the value  $d$  such that 95% of 45-54 year-old man has an FEV between  $4 - d$  and  $4 + d$ .

**Solution**  $X \sim N(4, 0.25)$ ,  $X = 0.5Z + 4$ , where  $Z \sim N(0, 1)$ .

- $Pr(X > 4.8) = Pr(0.5Z + 4 > 4.8) = Pr(Z > 1.6) = 0.0548$  (from column B)
- $Pr(X < 3.5) = Pr(0.5Z + 4 < 3.5) = Pr(Z < -1) = Pr(Z > 1) = 0.1587$  (from column B)
- (c)

$$\begin{aligned}0.95 &= Pr(4 - d < X < 4 + d) \\ &= Pr(4 - d < 4 + 0.5Z < 4 + d) \\ &= Pr(-2d < Z < 2d)\end{aligned}$$

From column D,  $Pr(-1.96 < Z < 1.96) = 0.95$ , so  
 $2d = 1.96 \Rightarrow d = 0.98$

Question 9 (4 points total) Assume that the number of episodes per year of otitis median, a common disease of the middle ear in early childhood, follows a Poisson distribution with parameter  $\lambda = 1.5$  episodes per year. Use TABLE 2 to answer the following questions (a), (b).

- (2 points) Find the probability of getting more than 2 episodes in the first year of life.
- (2 points) Find the probability of getting no episodes in the first two years of life.

**Solution** Let  $X(t)$  be the number of episodes per  $t$  years, then  $X(t) \sim \text{POI}(1.5t)$ .

(a)  $X(1) \sim \text{POI}(1.5)$

$$\begin{aligned} \Pr(X(1) > 2) &= 1 - \Pr(X(1) = 0) - \Pr(X(1) = 1) - \Pr(X(1) = 2) \\ &= 1 - 0.2231 - 0.3347 - 0.2510 = \mathbf{0.1912} \end{aligned}$$

(b)  $X(2) \sim \text{POI}(3)$ ,  $\Pr(X(2) = 0) = \mathbf{0.0498}$

**Question 10** (7 points total) A drug company in China is developing a new blood test to identify people with hepatitis B in China. The company used the blood test on 100 people in China who were known to have hepatitis B, and found out that 96 people in this group tested positive using the blood test. The company also used the blood test on 100 other people in China who were known not to have hepatitis B, and out of this group of people, 2 tested positive using the blood test.

(a) (2 points) What is the sensitivity of the test?

(b) (2 points) What is the specificity of the test?

(c) (3 points) What is the prevalence of hepatitis B in China if the predictive value negative  $PV^- = \frac{98}{99}$ .

**Solution**  $A$  = positive,  $B$  = pregnant,  $\Pr(A|B) = 0.96$ ,  $\Pr(A|\bar{B}) = 0.02$

(a) sensitivity =  $\Pr(A|B) = \mathbf{0.96}$

(b) specificity =  $\Pr(\bar{A}|\bar{B}) = 1 - \Pr(A|\bar{B}) = 1 - 0.02 = \mathbf{0.98}$

(c) prevalence =  $\Pr(B)$

$$\begin{aligned} PV^- &= \Pr(\bar{B}|\bar{A}) \\ &= \frac{\Pr(\bar{A}|\bar{B})\Pr(\bar{B})}{\Pr(\bar{A}|\bar{B})\Pr(\bar{B}) + \Pr(\bar{A}|B)\Pr(B)} \\ &= \frac{0.98(1 - \Pr(B))}{0.98(1 - \Pr(B)) + 0.04\Pr(B)} \\ &= \frac{98(1 - \Pr(B))}{98(1 - \Pr(B)) + 4\Pr(B)} \\ \frac{98}{99} &= \frac{98(1 - \Pr(B))}{98(1 - \Pr(B)) + 4\Pr(B)} \\ \frac{98}{1} &= \frac{98(1 - \Pr(B))}{4\Pr(B)} \\ \Pr(B) &= \mathbf{0.2} \end{aligned}$$