Math 21a Hourly 1
(1'st Practice Hourly)

What follows is a model of the first hourly exam (it is more or less the exam given last year in this course). The upcoming exam on Wednesday, October 10 will be roughly similar to this one in length and difficulty. To study for the upcoming exam, first work the suggested problems for the Chapters 1 and Appendix A. The exam will cover solely material from this portion of the text book. The list of suggested problems is posted on the course website. After you have worked the suggested problems and checked your answers with those in the book, try taking this exam as practice for the real thing. In this regard, note that you will have two hours for the real thing, but we hope that most people finish in well under the two hour limit. The answers to this practice hourly are provided at the website.

By the way, in the real exam, each problem will be printed on a separate page and you will be asked to provide your answers on that page and to use the back of that page if you need more space to show your work.

Remember that the exam on *Wednesday, October 10* is from *7-9pm* in *Lecture Halls C and D of the Science Center*. Please come a few minutes early as the exam will start promptly at 7pm.

Name: _____

Circle the name of your Section TF:

Arinkin(10)•Arinkin(12)•Bamberg•Comut•Kaplan•Karu•Knill•Libine•Liu•Taubes•Williams

Instructions:
- Print your name in the line above and circle the name of your section TA.
- Answer each of the questions below on the same page as the question. If more space is needed, use the back of the facing page. Extra blank pages are also provided at the back of this packet.
- Do not detach pages from this exam packet or unstaple this packet.
- Please write neatly. Answers which are deemed illegible by the grader will not receive credit.
- No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students. Use only your brain and a pencil.
- You have 2 hours to complete your work.
- Each problem counts 8 points.

By agreeing to take this exam, you are implicitly accepting Harvard University's honor code.

1. In the literary classic *Moby Dick* by Herman Melville, the captain of the whaling ship Pequod became obsessed with a particular white whale. The ship's log briefly described an encounter with this whale as follows: "In a coordinate system where the Pequod is at the origin, the ocean surface is the plane where $z = 0$ and $k = (0, 0, 1)$ points up. The whale swam on a straight line until it reached the surface; this line passed through the point $(-2, 8, -6)$ at time $t = 0$ and through the point $(1, 6, -4)$ at $t = 1$." No more about the encounter appeared in the log. Given the information just provided, answer the following questions:
   a) Find a parametric equation for the line that the whale traced as it swam along.
   b) As you know, whales are mammals so must surface to breath. What are the coordinates of the point where the whale surfaced for air?
   c) What is the closest distance that the whale came to the Pequod?

2. A bug is flying through the air and its position at time t is given by the end point of the vector
   $r(t) = (2 \ln(t), t^2, -2\sqrt{2}\, t)$.
   a) What is the bug's velocity at time t?
   b) What is the bug's speed at time t?

c) What is the length of the path of the bug between $t = 1$ and $t = 2$?

d) What is the component of the bug's position vector at time 1 in the direction of the vector $(1, 1, 1)$? (Express your answer as a vector.)

3. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are both non-zero vectors in space and set $\mathbf{w} = |\mathbf{u}| \, \mathbf{v} - |\mathbf{v}| \, \mathbf{u}$. Answer the following either true or false and give a short justification for your answer. (No points will be awarded without correct justification.)

a) The vector $\mathbf{w}$ is zero only when $\mathbf{u} = \mathbf{v}$.

b) $\mathbf{w}$ is perpendicular to $|\mathbf{u}| \, \mathbf{v} + |\mathbf{v}| \, \mathbf{u}$.

c) Assuming that $\mathbf{w}$ is not zero, then $\mathbf{w}$ is never parallel to either of the three coordinate axis.

4. Let Π denote the plane where $7x + 4y - 4z = 81$ and let L denote the line through $(1, 2, 4)$ and in the direction $\mathbf{v} = (12, -4, -4)$. Answer the following questions:

a) Do Π and L intersect? If so, at what point, and if not, why not?

b) Compute the distance from the origin to Π.

c) Π has a unit length normal vector, $\mathbf{n}$, whose dot product with $(1, 0, 1)$ is negative. What is the dot product of $\mathbf{v}$ with $\mathbf{n}$?

5. Introduce standard polar coordinates r and θ on the plane. Thus, $0 \le r < \infty$ and $0 \le \theta \le 2\pi$. The curve where $r = 2 \cos \theta$ is actually a circle in the plane. Find its center and radius.

6. Let Π denote the plane that contains the points $A = (1, 0, 0)$, $B = (0, 1, 0)$ and $C = (0, 0, 1)$. We say that a vector is 'tangent' to Π when the following happens: If some point on a line in the direction of this vector lies in Π then the whole line lies in Π. (Thus, with its starting point on Π, the whole vector lies in Π.)

a) Show that the vector $\mathbf{u} = (1, 2, -3)$ is tangent to Π.

b) Find a non-zero vector $\mathbf{v}$ which is tangent to Π and is also perpendicular to $\mathbf{u}$.

c) Let $\mathbf{w}$ be a vector which is tangent to Π. Explain why $\mathbf{w} - |\mathbf{u}|^{-2} \, (\mathbf{w} \cdot \mathbf{u}) \, \mathbf{u}$ must be proportional to the vector $\mathbf{v}$ that you just found in Part b).