Name: ________________________________

Math 21a Midterm 1 Wednesday, October 22nd 2003

Please circle your section:

Namhoon Kim Mauro Bruanstein MWF 9–10	Teru Yoshida Nathan Lange MWF 10–11	Tom Coates Barbara Richter MWF 10–11	Tom Coates Darin Ragozzine MWF 11–12
Jonathan Kaplan Chris Phillips MWF 11–12	Vivek Mohta Kory Byrns MWF 11–12		Vivek Mohta Ann Lai MWF 12–1
Abhinav Kumar Justin Ho Tu/Th 10–11:30	Sabin Cautis Shih En Lu Tu/Th 10–11:30		Andy Engelward Brad Burns T/Th 11:30–1

Question	Points	Score
1	18	
2	11	
3	17	
4	12	
5	16	
6	14	
7	12	
Total	100	

You have two hours to take this midterm. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You don't have to answer the problems in order – move on to another problem if you find that you're stuck and are spending too much time on one problem.

To receive full credit on a problem, you need to justify your answers carefully (unless the question specifically says otherwise). Unsubstantiated answers will receive little or no credit! Please be sure to write neatly – illegible answers will also receive little or no credit. If you're asked to provide a sketch of something, do so as neatly as possible – you could lose points if your sketch is difficult to understand.

If you need more space for a problem then use the back of the previous page to continue your work. Be sure to make a note of that so that the grader knows where to find your answers.

Please note that you are not allowed to use any notes or calculators during this test.

Good luck! Focus and do well!

Question 1. (18 points total)

Consider the plane $x + 3z = 1$, and the line parametrized by $x = 2t + 1, \quad y = 4t - 1, \quad z = 3t$

(a) (4 points) Find two different unit vectors that are normal to the plane.

(b) (2 points) Find a vector **a** that is parallel to the line.

(c) (3 points) Explain why the line is not parallel to the plane.

Question 1 continued

(Continue working with plane $x+3z=1$, and the line parametrized by $x=2t+1,\ \ y=4t-1,\ \ z=3t$)

(d) (3 points) Find the point of intersection of the line and the plane.

(e) (3 points) Find an equation for a plane that contains both the line and the origin.

(f) (3 points) Find the angle between the plane in part (e) and the original plane, $x+3z=1$ (recall that the angle between two planes is defined as the acute angle between their normal vectors). If you weren't able to find the plane in part (e), then use the plane $-2x+7y-12z=0$ instead.

Question 2. (11 points total) Let **a** and **b** be two vectors such that $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(a) (3 points) What is $(-3\mathbf{b}) \times (4\mathbf{a})$?

(b) (2 points) Find $\mathbf{a} \bullet (\mathbf{a} \times \mathbf{b})$ if it exists. If it doesn't exist then explain why not.

(c) (2 points) Find $\mathbf{a} \times (\mathbf{a} \bullet \mathbf{b})$ if it exists. If it doesn't exist then explain why not.

(d) (4 points) Suppose that not only does $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, but also that $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|$. Which of the following statements *must* be true? (simply circle all the statements that must be true – no explanations are necessary).

(i) $\mathbf{a} \neq \mathbf{b}$

(ii) **a** and **b** are parallel

(iii) **a** is perpendicular to **b**

(iv) either **a** or **b** must be a unit vector.

Question 3. (17 points total)

Consider the space curve defined by the vector function

$$\mathbf{r}(t) = \langle 3\cos(2t),\ \ 2t,\ \ 3\sin(2t) \rangle \quad \text{with } 0 \le t \le 2\pi$$

(a) (6 points) Sketch the curve as neatly as possible (be sure to label your axes).

(b) (3 points) Find an equation for a surface that this space curve lies on and describe the surface.

Question 3 continued

(Continue working with the space curve defined by $\mathbf{r}(t) = \langle 3\cos(2t),\ 2t,\ 3\sin(2t) \rangle$ with $0 \le t \le 2\pi$)

(c) (4 points) Find the length of this space curve.

(d) (4 points) Give parametric equations for the tangent line to this space curve at the point $(3, 2\pi, 0)$.

Question 4. (12 points total)

(a) (4 points) Consider the space curve that is defined as the intersection of the plane $y = z$ and the cylinder $x^2 + y^2 = 1$. Explain why this space curve can be parametrized by the vector function:

$$\mathbf{r}(t) = \langle \cos t, \sin t, \sin t \rangle \quad \text{with } 0 \le t \le 2\pi$$

(b) (4 points) Now consider the vector function $\mathbf{p}(t) = \dfrac{\mathbf{r}(t)}{|\mathbf{r}(t)|}$, where $\mathbf{r}(t)$ is the vector function given in part (a). Compute the components of vector function $\mathbf{p}(t)$.

(c) (4 points) Now consider the space curve formed by the intersection of the same plane $y = z$ and the unit sphere (the sphere of radius one centered at the origin). Explain why this new space curve is parametrized by the vector function $\mathbf{p}(t)$.

Question 5. (16 points total)

(a) (8 points) Sketch the graph of the function $f(x, y) = (y-1)^2 - 1$. Make your sketch as neat as possible, and be sure to label your axes carefully.

(b) (8 points) Now sketch the graph of the function $f(x, y) = \sqrt{x^2 + y^2 - 1}$. Again, make your sketch as neat as possible, and label your axes carefully.

Question 6. (14 points total)

A moth is flying around a lightbulb hanging from the ceiling of a room. The lightbulb is 10 feet above the floor and the moth stays exactly 10 feet above the floor as it flies around the lightbulb. The moth flies in a circular path, at constant speed, always exactly 2 feet away from the center of the lightbulb.

(a) (4 points) Write down the set of equations that describe the circle traced out by the moth in spherical coordinates. Consider the origin to be the point on the floor directly below the lightbulb.

(b) (4 points) Write down the equations giving the moth's circular path in cylindrical coordinates, and then give a parametrization of the moth's path in cylindrical coordinates.

(c) (6 points) Suppose that the moth's flight path is given by a vector function **r**(t), with parameter t representing time. Given what you know about the moth's flight answer the following questions:

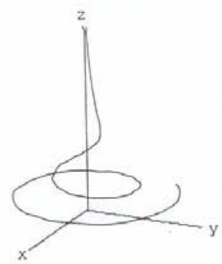
(i) Is it possible to determine **r'**(t) • **k**? If so, what is it?

(ii) Describe the curve parametrized by **r'**(t) x **k**

Question 7. (12 points total, 2 points each)

Round and round they go… Match the following space curves to the appropriate vector functions (and yes, we've been polite and made everything match up). Write the number of the appropriate space curve next to the correct vector function.

1.



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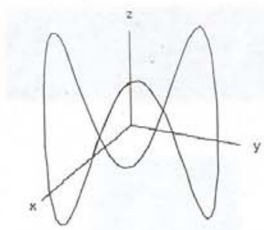

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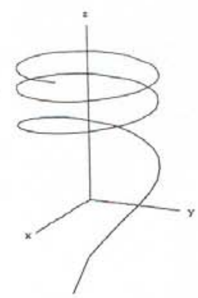

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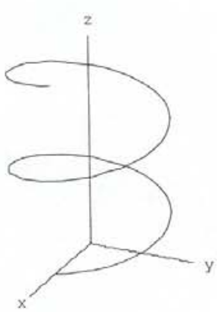

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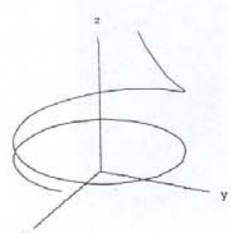
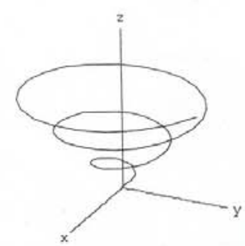


6.



____ $\mathbf{r}(t) = \langle \cos t, \sin t, \ln t \rangle$	____ $\mathbf{r}(t) = \left\langle t\cos t, t\sin t, \frac{1}{t} \right\rangle$	____ $\mathbf{r}(t) = \langle \cos t, \sin t, \sin(3t) \rangle$
____ $\mathbf{r}(t) = \langle t\cos t, t\sin t, t \rangle$	____ $\mathbf{r}(t) = \langle \cos t, \sin t, t) \rangle$	____ $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{1}{t} \right\rangle$