

**Math 21a Hourly 2**  
**Spring 1997**

**Instructions:**

- Please put your name and the name of your teaching fellow and your section time on your blue exam book. (If you use more than one book, put this information on each book used.)
- Answer as many of the following problems as you can. Remember to show all work in order to obtain partial credit where available. Unjustified answers may be disallowed.
- In working this exam, you are not allowed to use computers, calculators or any other aids save a pencil or pen.
- Write neatly and carefully. Illegible material will not be graded. Also, please start a new page in your blue exam book for each problem's answer.
- Note: Bold face type indicates vectors, regular type indicates scalars.
- Mind you that each problem below is worth the same amount, so do the easy ones first.

**Problems:**

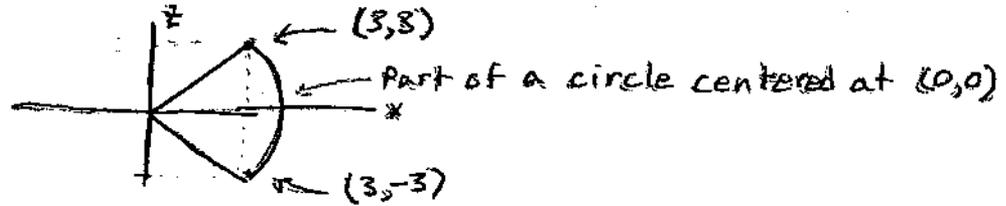
1. Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (z + \sqrt{x^2 + y^2}) dz dy dx$ .

2. Consider the following integral:

$$\int_0^4 \int_0^{2-\sqrt{y}} dx dy.$$

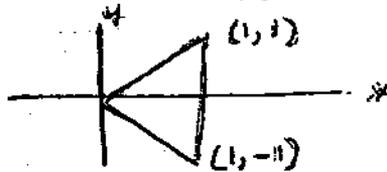
- a) Evaluate the integral.
  - b) Sketch the region of integration.
  - c) Write down the limits for the integral but where the y integration is done first instead.
3. Let  $f(x, y) = x^2 + 3y^2$  and let  $Q = (2, 1)$ .
- a) What direction does  $f$  increase most rapidly at  $Q$ ?
  - b) What is the directional derivative in that direction?
  - c) Find a vector  $\mathbf{n}$  which is tangent to the level curve of  $f$  through  $Q$ .
  - d) Let  $S$  denote the surface in 3-space where  $z = x^2 + 3y^2$ . Write a non-zero normal vector to this surface at the point  $(2, 1, 7)$  on  $S$ .
4. Suppose that the Alpha Widget Company sells  $A(K, L) = (3L + 60) K^{1/2}$  widgets per week with  $L$  widget makers and  $K$  sales people. The Beta Widget Company wishes to be more efficient. They find that with  $L$  widget makers and  $K$  sales people, they can usually sell  $B(K, L) = 2 K^{1/2} L^{1/2}$  widgets per week. By adjusting  $K$  and  $L$ , what is the most number of widgets per week that can be sold by Beta, given that Alpha sells 600 widgets per week with the same  $K$  and  $L$ ?

5. a) Find the critical value of the function  $f(x, y, z) = x^4 + y^2 - 2 \cdot zy + z^3 + 28$  in the plane where  $z = -3$ .  
 b) Give an argument to justify the assertion that this point is a minimum.
6. Maximize the function  $f(x, y, z) = x^{1/2} y^{1/4} z^{1/4}$  over the region where  $x \geq 0, y \geq 0, z \geq 0$  and  $2x + y + z \leq 4$ .
7. Rotate this shape around the z-axis:



If the density at any point in the solid is five times the square of the distance of the point to the  $xy$  plane, find the total mass of the solid.

8. Let  $R$  be the following region in the  $xy$  plane:



Find the volume of the set of points in 3-space above  $R$ , but where  $x^2 + z^2 \leq 1$ . (Thus, this set consists of those points  $(x, y, z)$  where  $(x, y)$  is in  $R, z > 0$ , and  $x^2 + z^2 \leq 1$ .)

9. Find the integral of the function  $f(x, y) = y^2 x$  over the region in the first quadrant of the  $xy$  plane where  $x$  is less than  $y$ , but where  $1 \leq x^2 + y^2 \leq 2$ .