3/14/2006, FIRST HOURLY SECOND PRACTICE **Math 21a, Spring 2006**

Name:

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MWF 11 Veronique Godin
TTH 10 Oliver Knill
TTH 115 Thomas Lam

- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which the grader can not read will not receive credit.
- No notes, books, calculators, computers, or other electronic aids can be used.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The vectors $\langle 3,-2,1\rangle$ and $\langle -6,4,-2\rangle$ are parallel.

2) T F If $|\vec{v}\times\vec{w}|=0$ then $\vec{v}=\vec{0}$ or $\vec{w}=\vec{0}$.

3) T F The surface $z^2+4y^2=x^2+1$ is a two sheeted hyperboloid.

4) T F The surface $4x^2-4x+y^2-2y-120=-z^2$ is an ellipsoid.

5) T F The parametrized lines $\vec{u}(t)=\langle 1+2t,2-5t,1+t\rangle$ and $\vec{v}(t)=\langle 3-4t,-3+10t,2-2t\rangle$ are the same line.

6) T F The surface $\sin(x)=z$ contains lines which are parallel to the y-axis.

7) T F If $\vec{u}\cdot\vec{v}=0$, $\vec{v}\cdot\vec{w}=0$ and $\vec{v}$ is not the zero vector, then $\vec{u}\cdot\vec{w}=0$.

8) T F The curvature of a curve depends upon the speed at which one travels upon it.

9) T F Two lines in space that do not intersect must be parallel.

10) T F The intersection of the ellipsoid $x^2/3+y^2/4+z^2/3=1$ with the plane $y=1$ is a circle.

11) T F The line $\vec{r}(t)=\langle 1+2t,1+2t,1-4t\rangle$ hits the plane $x+y+z=9$ at a right angle.

12) T F A line in space can intersect an elliptic paraboloid in 4 points.

13) T F There is a quadric which is a hyperbola when intersected with the plane $z=0$, which is a hyperbola when intersected with the plane $y=0$ and which is a parabola when intersected with $x=0$.

14) T F The vector $\vec{u}\times(\vec{v}\times\vec{w})$ is always in the same plane as $\vec{v}$ and $\vec{w}$.

15) T F If $\vec{u}\times\vec{v}=0$ and $\vec{u}\cdot\vec{v}=0$, then one of the vectors $\vec{u}$ and $\vec{v}$ is zero.

16) T F If the velocity vector $\vec{r}\,'(t)$ and the acceleration vector $\vec{r}\,''(t)$ of a curve are parallel at time $t=1$, then the curvature $\kappa(t)$ of the curve is zero at time $t=1$.

17) T F If the speed of a parametrized curve is constant over time, then the curvature of the curve $\vec{r}(t)$ is zero.

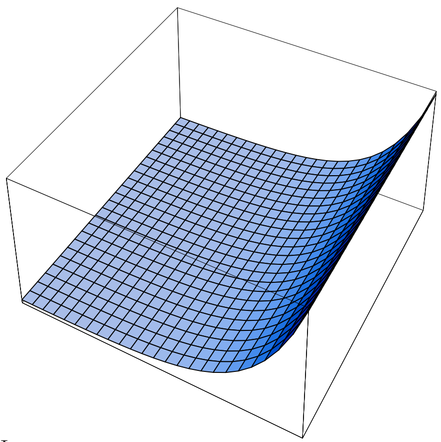
18) T F The scalar projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the scalar projection of $\vec{w}$ onto $\vec{v}$.

19) T F The value of the function $f(x,y)=\sqrt{1+3x+5y}$ at $(-0.002,0.01)$ can by linear approximation be estimated as $1-(3/2)\cdot 0.002+(5/2)\cdot 0.01$.

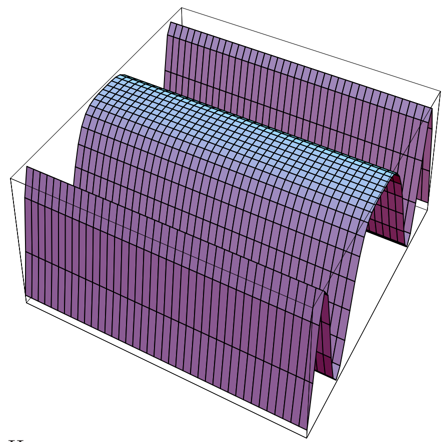
20) T F The function $f(x,y)=e^y x^2\sin(y^2)$ satisfies the partial differential equation $f_{xxyyyxyy}=0$.

Problem 2a) (2 points)

Match the equation with their graphs. No justifications are needed.



I



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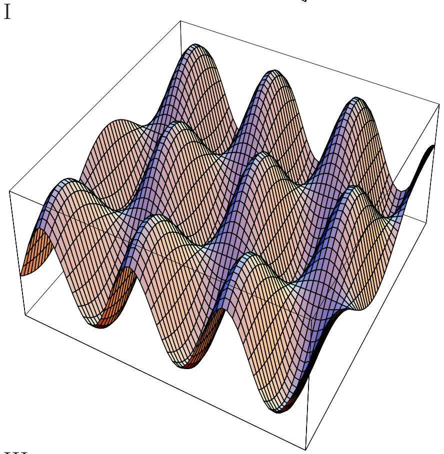

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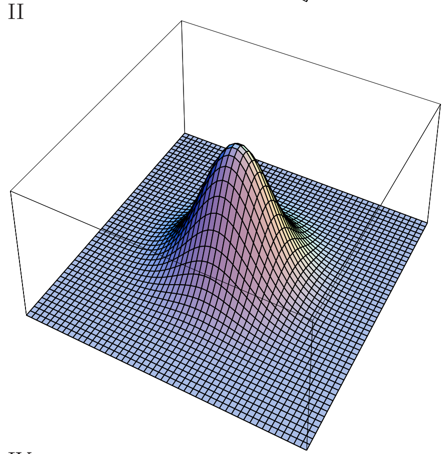
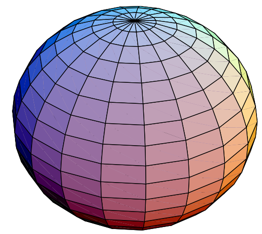


IV

Enter I,II,III,IV here	Equation
	$z = \sin(5x)\cos(2y)$
	$z = \cos(y^2)$
	$z = e^{-x^2-y^2}$
	$z = e^x$

Problem 2b) (3 points)

Match the parametric surfaces with their parameterization. No justification is needed.



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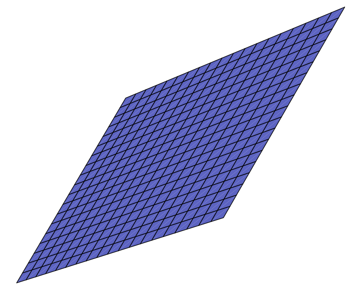

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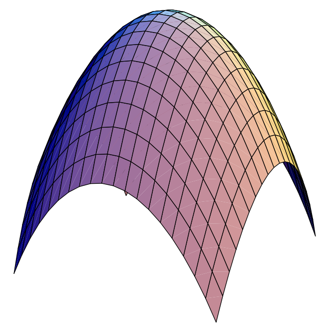

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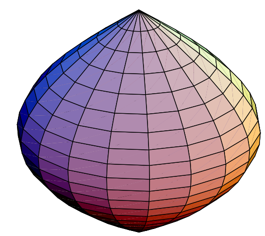


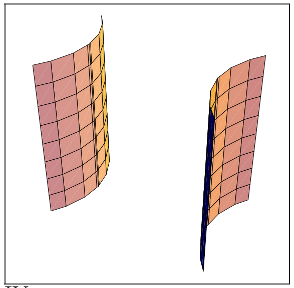
IV

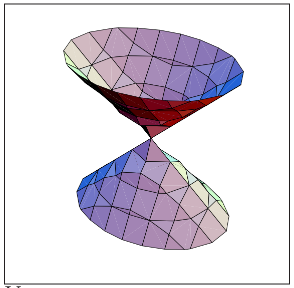
Enter I,II,III,IV here	Parameterization
	$(u,v) \mapsto (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$
	$(u,v) \mapsto (u-1, v+3, u+v)$
	$(u,v) \mapsto (u, v, 1-u^2-v^2)$
	$(u,v) \mapsto (\sin(v)\cos(u), \sin(v)\sin(u), v)$

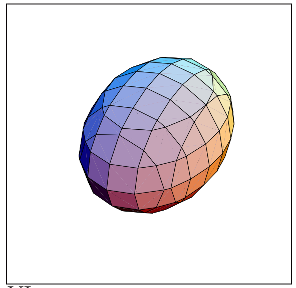
Problem 2c) (4 points)

Match the equations with the surfaces.

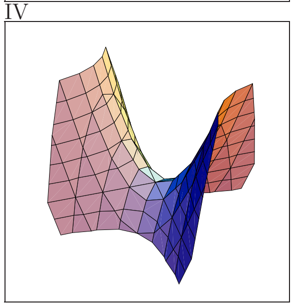
I II III



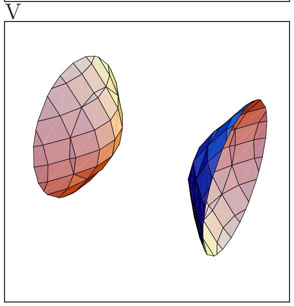




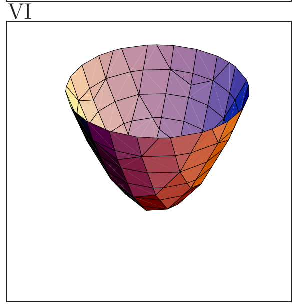
IV



V



VI



Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

Problem 3) (10 points)

a) Show that for any differentiable function $g(x)$, the function $u(x,y) = g(x^2+y^2)$ satisfies the partial differential equation $yu_x = xu_y$.

b) Assuming $g'(5) \neq 0$, let u be the function defined in a). Find the unit vector $\vec{v}$ in the direction of maximal increase at the point $(x,y) = (2,1)$.

Problem 4) (10 points)

a) (7 points) Find a parametric equation for the line which is the intersection of the two planes $2x - y + 3z = 9$ and $x + 2y + 3z = -7$.

b) (3 points) Find a plane perpendicular to both planes and which passes through the point $P = (1,1,1)$.

Problem 5) (10 points)

Given the vectors $\vec{v} = \langle 1,1,0 \rangle$ and $\vec{w} = \langle 0,0,1 \rangle$ and the point $P = (2,4,-2)$. Let Σ be the plane which goes through the origin which contains the vectors $\vec{v}$ and $\vec{w}$. Let S be the unit sphere $x^2 + y^2 + z^2 = 1$.

a) (6 points) Compute the distance from P to the plane Σ.

b) (4 points) Find the shortest distance from P to the sphere S.

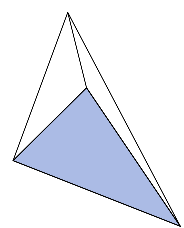
Hint for b): Find first the distance from P to the origin $O = (0,0,0)$.

Problem 6) (10 points)

a) (6 points) Find an equation for the plane through the points $A = (0,1,0), B = (1,2,1)$ and $C = (2,4,5)$.

b) (4 points) Given an additional point $P = (-1,2,3)$, what is the volume of the tetrahedron which has A, B, C, P among its vertices.

A useful fact which you can use without justification in b): the volume of the tetrahedron is 1/6 of the volume of the parallelepiped which has AB, AC, and AP among its edges.



Problem 7) (10 points)

The parametrized curve $\vec{u}(t) =< t, t^2, t^3 >$ (known as the "twisted cubic") intersects the parametrized line $\vec{v}(s) = \langle 1+3s, 1-s, 1+2s \rangle$ at a point P. Find the angle of intersection.

Problem 8) (10 points)

Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (\log(t), 2t, t^2)$, where $\log(t)$ is the natural logarithm (denoted by $\ln(t)$ in some textbooks).

a) What is the velocity and what is the acceleration at time $t = 1$?

b) Find the length of the curve from $t = 1$ to $t = 2$.

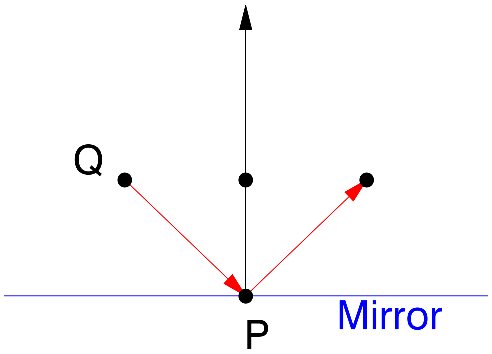
Hint: you should end up with a final integral which does not involve any square roots and which you can solve.

Problem 9) (10 points)

A planar mirror in space contains the point $P = (4, 1, 5)$ and is perpendicular to the vector $\vec{n} = \langle 1, 2, -3 \rangle$. The light ray $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$ with source $Q = (7, 0, 7)$ hits the mirror plane at the point P.

a) (4 points) Compute the projection $\vec{u} = \vec{\text{Proj}}_{\vec{n}}(\vec{v})$ of $\vec{v}$ onto $\vec{n}$.

b) (6 points) Identify $\vec{u}$ in the figure and use it to find a vector parallel to the reflected ray.



Problem 10) (10 points)

a) A duck swims near Watertown on the Charles river clockwise on the circle $\vec{r}(t) = \langle \cos(t), -\sin(t) \rangle$. The water temperature is given by the formula $T(x, y) = x^3 e^y + y$. Find the the temperature change $\frac{dT}{dt}$ the duck feels at time $t = 0$

b) The place where the temperature $T(x, y) = 1$ is constant can be writen as $y = f(x)$ near the point $(1, 0)$. Find $f'(1)$.